

MATH 100 – WORKSHEET 22
TAYLOR POLYNOMIALS

1. TAYLOR EXPANSION OF e^x

Let $f(x) = e^x$

- (1) Find $f(0), f'(0), f^{(2)}(0), \dots$
- (2) Find a simple polynomial $T_0(x)$ such that $T_0(0) = f(0)$.
- (3) Find a simple polynomial $T_1(x)$ such that $T_1(0) = f(0)$ and $T_1'(0) = f'(0)$.
- (4) Find a simple polynomial $T_2(x)$ such that $T_2(0) = f(0), T_2'(0) = f'(0)$ and $T_2^{(2)}(0) = f^{(2)}(0)$.
- (5) Find a simple polynomial $T_3(x)$ such that $T_3^{(k)}(0) = f^{(k)}(0)$ for $0 \leq k \leq 3$.

2. TAYLOR EXPANSION OF \sqrt{x} ABOUT $x = 4$

Let $f(x) = \sqrt{x}$

- (1) Find a simple polynomial $T_0(x)$ such that $T_0(4) = f(4)$.
- (2) Find a simple polynomial $T_1(x)$ such that $T_1(4) = f(4)$ and $T_1'(4) = f'(4)$.
- (3) Find a simple polynomial $T_2(x)$ such that $T_2(4) = f(4), T_2'(4) = f'(4)$ and $T_2^{(2)}(4) = f^{(2)}(4)$.
- (4) Find a simple polynomial $T_3(x)$ such that $T_3^{(k)}(4) = f^{(k)}(4)$ for $0 \leq k \leq 3$.

3. GENERAL FORMULA

The n th order Taylor expansion of $f(x)$ about $x = a$ is the polynomial

$$T_n(x) = c_0 + c_1(x - a) + \cdots + c_n(x - a)^n$$

where $c_k = \frac{f^{(k)}(a)}{k!}$.

(1) Find the 4th order expansion of $\frac{1}{1-x}$

(2) Find the n th order expansion of $\sin x$.

4. NEW FROM OLD

(1) Find the 3rd order Taylor expansion of $\sqrt{4+x}$ about $x = 0$.

(2) Find the 3rd order Taylor expansion of $\sqrt{4+x} + \frac{1}{1-x}$ about $x = 0$.

(3) Find the 8th order Taylor expansion of $e^{x^2} + \sin(5x)$

(4) Find the 3rd order Taylor expansion of $e^{\sin x} \cdot \cos(\sqrt{x})$.