MATH 100 – WORKSHEET 22
ESTIMATES ON TAYLOR EXPANSIONS

The Taylor expansion of $f(x)$ about $x = a$ is

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

Then there is c between a and x such that

$$R_n(x) = f(x) - T_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

Moral: The remainder looks like the next term except the derivative is evaluated at the point c.

1. Linear approximation of $(1001)^{1/3}$

(1) Estimate $(1001)^{1/3}$ using a linear approximation. Express your answer as a rational number.
(2) Write down the remainder term as it applies to this case. In which range does c vary?
(3) Give an upper bound for the magnitude of the error in your approximation. What is the sign?

2. Taylor expansion of e^x

Let $f(x) = e^x$ and recall that the Maclaurin expansion is $T_n(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$.

(1) Estimate e using a second order Taylor expansion. Write your answer as a rational number.
(2) Estimate the error.
(3) Repeat for $\frac{1}{e}$.

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3. Taylor expansion of $\sqrt{x}$ about $x = 4$

Let $f(x) = \sqrt{x}$ and recall that about $a = 4$ we have $T_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$ and that $f^{(3)}(x) = \frac{3}{8x^{5/2}}$.

(1) Approximate $\sqrt{5}$ using a 2nd order expansion.
(2) Bound the error in your expansion.
(3) Approximate $\sqrt{5}$ using a 3rd order expansion.
(4) Bound the error in your approximation.