MATH 100 – WORKSHEET 26
MINIMA AND MAXIMA, MVT

1. More Minima and Maxima

(1) Find the critical numbers of $f(x) = \begin{cases} x^3 - 6x^2 + 3x & x \leq 3 \\ \sin(2\pi x) - 18 & x \geq 3 \end{cases}$.

(2) Find the absolute minimum and maximum of $g(x) = xe^{-x^2/8}$ on
   (a) $[-1, 4]$
   (b) $[0, \infty)$

(3) Show that the function $3x^3 + 2x - 1 + \sin x$ has no local maxima or minima.

Date: 2/11/2012.

2. The Mean Value Theorem

Theorem. *Let f be defined differentiable on $[a,b]$. Then there is $a < c < b$ such that $\frac{f(b)-f(a)}{b-a} = f'(c)$. Equivalently, for any x there is c between a, x so that $f(x) = f(a) + f'(c)(x-a)$.*

(1) Let $f(x) = e^x$ on the interval $[0,1]$. Find all values of c so that $f'(c) = \frac{f(1)-f(0)}{1-0}$.

(2) Let $f(x) = |x|$ on the interval $[-1,2]$. Find all values of c so that $f'(c) = \frac{f(2)-f(-1)}{2-(-1)}$

(3) Suppose that $f'(x) > 0$ for all x. Show that $f(b) > f(a)$ for all $b > a$. (Hint: consider the sign of $\frac{f(b)-f(a)}{b-a}$).

(4) Show that $f(x) = 3x^3 + 2x - 1 + \sin x$ has exactly one real zero.

Corollary (Monotone function test)**.** *Let f be a function such that f' exists and is continuous on $[a,b]$. Suppose that $f'(x) \neq 0$ for $a < x < b$. Then f has an inverse function on this interval.*