MATH 100 – WORKSHEET 27
MORE MVT

1. The Mean Value Theorem

Theorem. *Let f be defined differentiable on $[a,b]$. Then there is $a < c < b$ such that $\frac{f(b)-f(a)}{b-a} = f'(c)$. Equivalently, for any x there is c between a, x so that $f(x) = f(a) + f'(c)(x-a)$.*

(1) Let $f(x) = |x|$ on the interval $[-1,2]$. Find all values of c so that $f'(c) = \frac{f(2)-f(-1)}{2-(-1)}$

(2) Suppose that $f'(x) > 0$ for all $a < x < b$. Show that f is strictly increasing in $[a,b]$. (Hint: consider the sign of $\frac{f(b)-f(a)}{b-a}$).

(3) Show that $f(x) = 3x^3 + 2x - 1 + \sin x$ has exactly one real zero. (Hint: let a, b be zeroes of f. The MVT will find c such that $f'(c) = ?$)

Corollary (Monotone function test)**.** *Let f be a function such that f' exists and is continuous on $[a,b]$. Suppose that $f'(x) \neq 0$ for $a < x < b$. Then f has an inverse function on this interval.*

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(1) Show that $|\sin a - \sin b| \leq |a - b|$ for all a, b.

(2) Let $x > 0$. Show that $e^x > 1 + x$ and that $\ln(1 + x) \leq x$.