MATH 253 – COMMON ERRORS IN THE FIRST MIDTERM EXAM

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1. Q1: Partial Differentiation

- When differentiating $f(x,y) = x^2 \int_0^y \sin^8\left(\sqrt{t}\right) \mathrm{d}t$
  - This is a product of x^2 (a function of x along) and $\int_0^y \sin^8\left(\sqrt{t}\right) \mathrm{d}t$ (a function of y alone). So when we differentiate with respect to x we get $2x \int_0^y \sin^8\left(\sqrt{t}\right) \mathrm{d}t$. Writing $2x\sin^8(\sqrt{y})$ is a clear error.
  - When differentiating with respect to y, a common confusion was to differentiate $\sin^8\left(\sqrt{t}\right)$ with respect to t. The function of y is actually the integral, and the fundamental theorem of calculus says $\frac{d}{dy}\int_0^y F(t)dt = F(y)$, not $\frac{d}{dy}\int_0^y F(t)dt = F'(y)$!.
- When differentiating $z + e^z = x^2 + y^2$.
  - You can't solve for z! Writing $z = x^2 + y^2 - e^z$ doesn't help because there's an e^z on the RHS.
  - After you get $z_x = \frac{2x}{1+e^z}$ you want to differetiate wrt y. It's true that $2x$ is constant for that, but $\frac{1}{1+e^z}$ isn't – z still depends on y! thus the derivative is

$$\begin{aligned}\frac{\partial}{\partial y}\frac{\partial z}{\partial x} &= \frac{\partial}{\partial y}\frac{2x}{1+e^z} \overset{\text{chain rule}}{=} 2x\cdot\left(-\left(1+e^z\right)^{-2}\right)\frac{\partial e^z}{\partial y} \\ &\overset{\text{chain rule}}{=} -\frac{2x}{(1+e^z)^2}e^z\frac{\partial z}{\partial y} \\ &\overset{\text{done before}}{=} -\frac{2x}{(1+e^z)^2}e^z\frac{2y}{(1+e^z)} = -\frac{4xye^z}{\left(1+e^z\right)^3}\,.\end{aligned}$$

2. Q3: Tangent planes and linear approximation

- Some people still used the (incorrect) non-linear approximation $f(x_0,y_0) + f_x\left(x,y\right)\left(x - x_0\right) + f_y\left(x,y\right)\left(x - x_0\right)$ instead of the correct linear approximation

$$f(x,y) \approx f_x\left(x_0,y_0\right) + f_x\left(x_0,y_0\right)\left(x - x_0\right) + f_y\left(x_0,y_0\right)\left(x - x_0\right).$$

  Note that writing the first approximation as $f(x_0,y_0) + f_x\left(x,y\right)\Delta x + f_y\left(x,y\right)\Delta y$ and then plugging in (x_0,y_0) for (x,y) and some value of $\Delta x, \Delta y$ accidentally gives the right value for the linear approximation, but makes no sense.

3. Q4: Geometry

- The line through the points D, E has the *parametrization* $D + t\overrightarrow{DE}$. If $\overrightarrow{DE} = \langle a,b,c\rangle$ then a geometric object with the equation $ax + by + cz = d$ is a plane perpendicular to $\overrightarrow{DE}$, not a line.
- On the angle between a line and a plane.
  - The angle between a line and a plane is not the same as the angle between the line and any vector in the plane – for different vectors in the plane you will get different angles.
  - We defined angles between lines to be *acute*. So if you find that the angle is $\cos^{-1}\left(-\frac{1}{2}\right)$ you should have reversed one of your vectors and obtained the angle $\cos^{-1}\left(\frac{1}{2}\right)$ instead.
  - The angle between the line and the vector *normal* to the plane is not the same as the angle with the plane - in fact the two angles add to $\frac{\pi}{2}$.