MATH 253 – WORKSHEET 13
THE CHAIN RULE

(1) Define z as a function of x, y as the solution to $2x + 3y - 4z - e^{xyz-1} = 0$.
   (a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

   (b) Find the plane tangent to this surface at $(1, 1, 1)$.

   (c) Find an approximate solution to $\frac{5}{3} + \frac{7}{2} - 4z - e^{\frac{35}{36}z-1} = 0$.

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(2) Suppose that $w = x^2 + yz - \ln(1+z)$, that $x = st$, that $y = s + t$ and that $z = \frac{s}{t}$. Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$.

(3) Suppose that z is a function of x, y and that x, y are functions of r, θ according to $x = r\cos\theta$, $y = r\sin\theta$. Express $\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$ in terms of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(4) You are driving at a constant speed on a road that keeps a fixed compass direction as it goes over a hill. Say you position at time t is $(1-t, t)$, and hill is described by $z = e^{-x^2-y^2}$. How fast is your elevation changing at time t? When is your elevation maximal? What is it then?