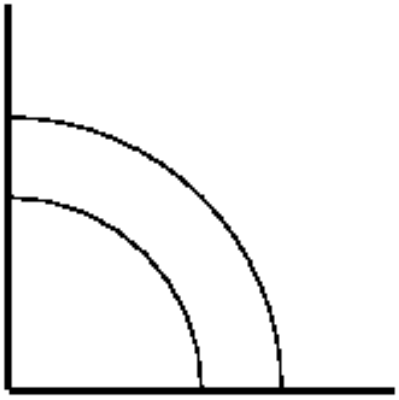
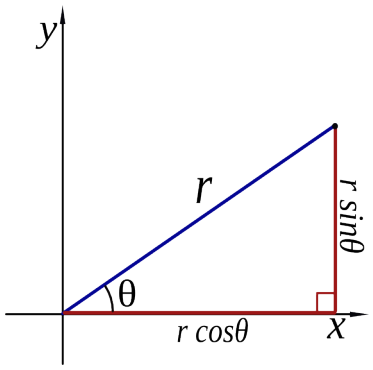
# MATH 253 – WORKSHEET 23
# POLAR COORDINATES AND INTEGRATION

## 1. Polar coordinates

Given $(x, y)$ set $r = \sqrt{x^2 + y^2}$, $\theta = \arctan\left(\frac{y}{x}\right)$. Given $(r, \theta)$ set $x = r\cos\theta$, $y = r\sin\theta$.






(1) Let $D = \{(x, y) \mid 1 \le x^2 + y^2 \le 2,\ x, y \ge 0\}$.
   (a) Express $D$ in the form $D = \{(r, \theta) \mid ??\}$

   (b) Try expressing $\iint_D \cos\left(x^2 + y^2\right) \mathrm{d}A$ as an iterated integral, slicing the domain vertically.

   (c) Calculate $\iint_D \cos\left(x^2 + y^2\right) \mathrm{d}A$ in polar coordinates.

*Date*: 1/11/2013.

(2) Find the volume of the solid lying above the $xy$-plane, below the paraboloid $z = x^2 + y^2$ and inside the cylinder $(x-1)^2 + y^2 = 1$.

(a) Find a region $R$ in the plane and a function $f(x, y)$ so that the volume is $\iint_R f(x, y)\,\mathrm{d}A$.

(b) Write $R$ and $f$ in polar coordinates.

(c) Evaluate the integral.

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