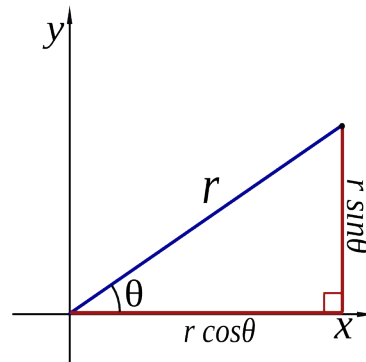
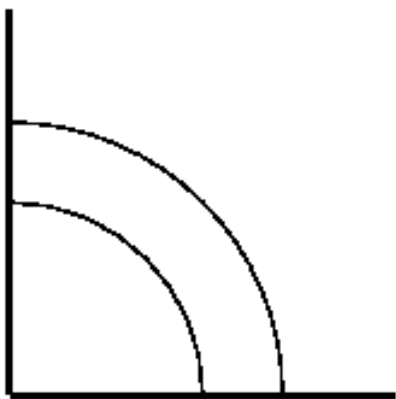


**MATH 253 – WORKSHEET 23**  
**POLAR COORDINATES AND INTEGRATION**

1. POLAR COORDINATES

Given  $(x, y)$  set  $r = \sqrt{x^2 + y^2}$ ,  $\theta = \arctan\left(\frac{y}{x}\right)$ . Given  $(r, \theta)$  set  $x = r \cos \theta$ ,  $y = r \sin \theta$ .



- (1) Let  $D = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 2, x, y \geq 0\}$ .  
(a) Express  $D$  in the form  $D = \{(r, \theta) \mid \text{??}\}$

(b) Try expressing  $\iint_D \cos(x^2 + y^2) \, dA$  as an iterated integral, slicing the domain vertically.

(c) Calculate  $\iint_D \cos(x^2 + y^2) \, dA$  in polar coordinates.

(2) Find the volume of the solid lying above the  $xy$ -plane, below the paraboloid  $z = x^2 + y^2$  and inside the cylinder  $(x - 1)^2 + y^2 = 1$ .

(a) Find a region  $R$  in the plane and a function  $f(x, y)$  so that the volume is  $\iint_R f(x, y) \, dA$ .

(b) Write  $R$  and  $f$  in polar coordinates.

(c) Evaluate the integral.

---

(Image credit: user Met501 and others on Wikipedia; see [http://en.wikipedia.org/wiki/File:Polar\\_to\\_cartesian.svg](http://en.wikipedia.org/wiki/File:Polar_to_cartesian.svg). Accordingly, this PDF is published under the CC Attribution-ShareAlike-Unported 3.0 license)