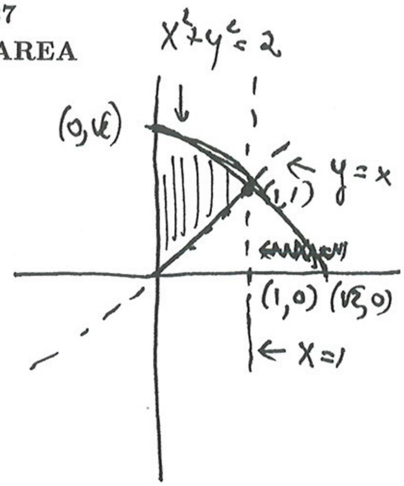
$\sin(a+b) \neq \sin(a)+\sin(b)$, $\frac{1}{a+b} \neq \frac{1}{a}+\frac{1}{b}$

$(a+b)^2 \neq a^2+b^2$, $\sqrt{a+b} \neq \sqrt{a}+\sqrt{b}$

MATH 253 – WORKSHEET 27
MIDTERM REVIEW, SURFACE AREA



(1) Evaluate $\int_{x=0}^{x=1} \mathrm{d}x \int_{y=1-\sqrt{1-x^2}}^{y=1+\sqrt{1-x^2}} \mathrm{d}y\, x \sin\left(\pi\left(1-y^2+\frac{y^3}{3}\right)\right)$.

Domain: $y \le 1+\sqrt{1-x^2}$ so $y^2 \le 1+(1-x^2) = 2-x^2$

$y \ge 1-\sqrt{1-x^2}$ so $y^2 \ge 1-(1-x^2) = x^2$

$(a-b)^2 \neq a^2-b^2$

so $x^2+y^{2} \le 2$ ← missing

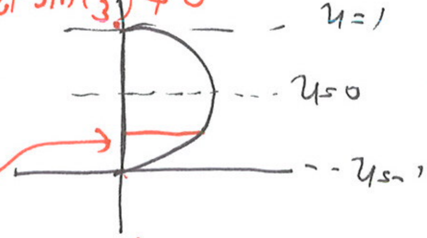
$y \ge x$

possibly $y<0$, $|y| \ge |x|$

Evaluation 1: Integrand is odd in x, so $=0$ ← integrand is odd, but domain not symmetric ($x \ge 0$)

Evaluation 2: Say $y=1+u$, so $\sin(\pi(1-y^2+y^3/3)) = \sin\left(\pi\left(1-(1+u)^2+\frac{(1+u)^3}{3}\right)\right)$ is odd in u, and domain is symmetric $u \leftrightarrow -u$. $\Rightarrow$ integral $=0$

no: $\sin(-\theta) = -\sin(\theta)$, but this isn't the same: set $u=0$, get $\sin(\frac{\pi}{3}) \neq 0$



Evaluation 3: Switch order of integration, get:

no symmetry →

$$\cancel{2} \int_{y=0}^{2\,\cancel{1}} dy \int_{x=\sqrt{1-(\cancel{1-y})^2}\ 0}^{x=\sqrt{1-(y-1)^2}} dx\ x\sin(\pi(1-y^2+y^3/3)) =$$

lower bound = upper bound !!! alarm

$$= 2\int_{y=0}^{y=1} dy \sin(\pi(1-y^2+y^3/3)) \underbrace{\left[\frac{x^2}{2}\right]_{\sqrt{1-(1-y)^2}}^{\sqrt{1-(y-1)^2}}}_{=0\ (\text{upper} = \text{lower})} = 2\int_{y=0}^{1} \sin\left(\pi\left(1-y^2+\frac{y^3}{3}\right)\right)(2y-y^2)\,dy =$$

$$= -2\int_{y=0}^{y=1} \sin(\pi u)\,du = 2\int_{u=\cancel{1/3}\ 1}^{u=\cancel{1}\ 1/3} \sin(\pi u)\,du = \frac{2}{\pi}\cancel{2\pi}\left[-\cos(\pi u)\right]_{u=\frac{1}{3}}^{u=1} = 2\pi\left[\cos(\pi) - \cos\left(\frac{\pi}{3}\right)\right]$$

$u = 1-y^2+y^3/3$

$du = (y^2-2y)\,dy$

bounds were switched

$$= 2\pi \cdot \left(-\frac{1}{2}\right) = \pi$$

$\cos \pi = (-1)$

Date: 15/11/2013.