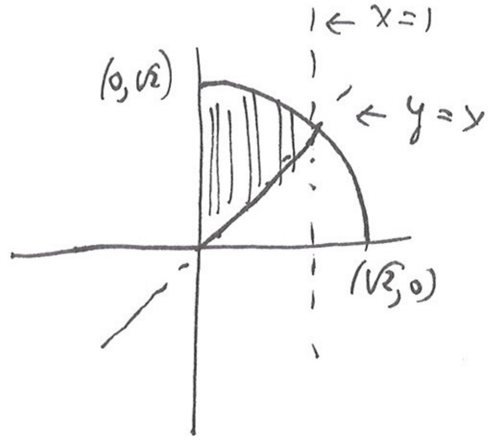
Find All Erros Below:

MATH 253 – WORKSHEET 27
MIDTERM REVIEW

(1) Evaluate $\int_{x=0}^{x=1} \mathrm{d}x \int_{y=1-\sqrt{1-x^2}}^{y=1+\sqrt{1-x^2}} \mathrm{d}y\, x \sin\left(\pi\left(1 - y^2 + \frac{y^3}{3}\right)\right)$.



Domain: $y \le 1+\sqrt{1-x^2}$ so $y^2 \le 1+(1-x^2) = 2-x^2$
$y \ge 1-\sqrt{1-x^2}$ $\quad y^2 \ge 1-(1-x^2) = x^2$

so $x^2+y^2 \le 2$
$y \ge x$

Evaluation 1: Integrand is odd in x, so $=0$

Evaluation 2: Say $y=1+u$, so $\sin(\pi(1-y^2+y^3/3)) = \sin\left(\pi\left(1-(1+u)^2+\frac{(1+u)^3}{3}\right)\right)$ is odd in u, and domain is symmetric about $u=0$ $\Rightarrow$ integral $=0$

Evaluation 3: Switch order of integration, get:

$$2\int_{y=0}^{y=1} dy \int_{x=\sqrt{1-(1-y)^2}}^{x=\sqrt{1-(y-1)^2}} dx \; x\sin\left(\pi\left(1-y^2+y^3/3\right)\right) =$$

$$= 2\int_{y=0}^{1} dy \sin\left(\pi\left(1-y^2+y^3/3\right)\right)\left[x^2\right]_{\sqrt{1-(1-y^2)}}^{\sqrt{1-(y-1)^2}} = 2\int_{y=0}^{1} \sin\left(\pi\left(1-y^2+y^3/3\right)\right)\left(2y-y^2\right)dy =$$

$$= 2\int_{y=0}^{1} \sin(\pi u)\,du = 2\int_{u=1/3}^{u=1} \sin(\pi u)\,du = 2\pi\left[\cos(\pi u)\right]_{u=1/3}^{u=1} = 2\pi\left[\cos(\pi) - \cos\left(\frac{\pi}{3}\right)\right]$$

$u = 1-y^2+y^3/3$

$$= 2\pi\left(-\frac{1}{2}\right) = \pi$$

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