MATH 253 – WORKSHEET 32
SPHERICAL COORDINATES

We replace (x, y, z) with (r, θ, ϕ) via:

$$x = \rho \sin\phi \cos\theta \qquad y = \rho \sin\phi \sin\theta \qquad z = \rho \cos\phi$$
$$\rho = \sqrt{x^2 + y^2 + z^2} \qquad \tan\theta = \tfrac{y}{x} \qquad \cos\phi = \frac{z}{\rho}$$

The volume element is

$$\mathrm{d}V = \rho^2 \sin\phi \, \mathrm{d}\rho \, \mathrm{d}\theta \, \mathrm{d}\phi \, .$$

(1) Express the following surfaces in spherical coordinates.
   (a) The sphere of radius 2 about the origin.

   (b) The "double cone" $z^2 = x^2 + y^2$.

   (c) The paraboloid $z = x^2 + y^2$.

(2) Let B be the ball of radius 1 about the origin. Evaluate $\iiint_B e^{-\left(x^2+y^2+z^2\right)^{3/2}} \, \mathrm{d}V$.

(3) Describe the following regions in words, then set up integration in spherical coordinates:
   (a) $E = \left\{(x, y, z) \mid x, y, z \geq 0,\ x^2 + y^2 + z^2 \leq 9\right\}$
   (b) $E = \left\{(x, y, z) \mid x^2 + y^2 + (z-1)^2 \leq 1\right\}$

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Cylindrical or Spherical?

(1) Let E be the "dimple" inside the sphere $x^2 + y^2 + z^2 = 2$ and above the paraboloid $z = x^2 + y^2$. Set up integration on it in spherical and cylindrical coordinates.

(2) Let E be the region above the cone $3z = \sqrt{x^2 + y^2}$ and below the plane $z = \frac{1}{2}$. Set up integration on it.