

MATH 100 – WORKSHEET 3
CONTINUITY

1. CONTINUITY

(1) Which of these functions are continuous? Why?

(a) $f(x) = \begin{cases} x & x < 0 \\ \cos x & x \geq 0 \end{cases}$

(b) $f(x) = \begin{cases} x & x < 0 \\ \sin x & x \geq 0 \end{cases}$

(2) Let $f(x) = \frac{x^3 - x^2}{x - 1}$.

(a) Why is $f(x)$ discontinuous at $x = 1$?

(b) Find b such that $g(x) = \begin{cases} f(x) & x \neq 1 \\ b & x = 1 \end{cases}$ is continuous everywhere.

(3) Find c, d such that $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ c & x = 1 \\ d - x^2 & x > 1 \end{cases}$ is continuous.

(4) Where are the following functions continuous?

(a) $\frac{x^2 + 2x + 1}{2 + \cos x}$

(b) $\frac{2 + \cos x}{x^2 + 2x + 1}$

(c) $\log((\sin x)^2)$

(Final 2011) Suppose f, g are continuous such that $g(3) = 2$ and $\lim_{x \rightarrow 3} (xf(x) + g(x)) = 1$. Find $f(3)$.

2. THE INTERMEDIATE VALUE THEOREM

Theorem. *Let $f(x)$ be continuous for $a \leq x \leq b$. Then $f(x)$ takes every value between $f(a), f(b)$.*

(1) Show that:

(a) $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \leq x \leq 1$.

(b) $\cos x = x$ has a solution.