

MATH 100 – WORKSHEET 4
CONTINUITY, HORIZONTAL ASYMPTOTES, THE DERIVATIVE

1. THE INTERMEDIATE VALUE THEOREM

(1) Show that:

(a) $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \leq x \leq 1$.

(b) There is $x > 0$ for which $\frac{1}{x} = \sin x$.

(2) (Final 2011) Let $y = f(x)$ be continuous with domain $[0, 1]$ and range in $[3, 5]$. Show the line $y = 2x + 3$ intersects the graph of $y = f(x)$ at least once.

2. HORIZONTAL ASYMPTOTES

(1) Evaluate the following limits:

(a) $\lim_{x \rightarrow \infty} \frac{x^2+1}{x-3}$

(b) $\lim_{x \rightarrow \infty} \frac{x^2+8}{2x^3-1}$

(c) $\lim_{x \rightarrow \infty} \frac{\sqrt{x^4+\sin x}}{x^2-\cos x}$

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(d) $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + 2x} - \sqrt{x^2 - 1})$

(2) Find the horizontal and vertical asymptotes of $\frac{x^2+x+1}{x^2-4}$

3. CALCULATE THE DERIVATIVES

Definition. $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

- (1) Find $f'(a)$ if
- (a) $f(x) = x^2$, $a = 3$.
 - (b) $f(x) = x^2$.
 - (c) $f(x) = \frac{1}{x}$.