Math 101 – WORKSHEET 31
MANIPULATING POWER SERIES

1. Manipulating power series: Geometric Series

Recall that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$.

(1) Find a power series representation for
   (a) (Final 2014) $\frac{x^3}{1-x}$
   (b) (Final 2011) $\frac{1}{1+x^3}$

(2) Find a power series representation for $\frac{1}{x+3}$
   (a) Expanding about $a = 0$
   (b) Expanding about $a = 7$

2. Manipulating power series: Calculus

(3) (Final 2011) Evaluate the following indefinite integral as a power series, and find its radius of convergence: $\int \frac{\mathrm{d}x}{1+x^3}$

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(4) Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $g(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} x^n$. Last time we verified that f converges everywhere, while g converges for $-1 < x \leq 1$.

(a) Find the power series representation of $f'(x)$. What is $f(x)$?

(b) Find the power series representation of $g'(x)$. What is $g'(x)$? What is $g(x)$?

(c) Conclude that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \log 2$.

(d) Find the power series representation of $\int_0^x \exp(-t^2)\, dt$.

3. Manipulating power series: summing series

(5) Evaluate $\sum_{n=1}^{\infty} \frac{n}{2^n}$.