14. Partial Fractions II (3/2/2017)

Goals. (no worksheet)

(1) Partial fractions:
  (a) Review: Form of the expansion
  (b) Polyomial division and the rational root theorem
  (c) Reduction of numerators
(2) Quiz

Last time: Partial fractions: Started with $\frac{5x+3}{(x+2)(2x-3)}$

(1) "Form": expect $\frac{5x+3}{(x+2)(2x-3)} = \frac{A}{x+2} + \frac{B}{2x-3}$

In general: If have $\frac{f(x)}{(x-b_1)^{d_1}(x-b_2)^{d_2}\cdots(x-b_r)^{d_r}} =$ $\deg f < \deg(\text{denom})$

$$= \frac{A_1}{(x-b_1)^{d_1}} + \frac{A_2}{(x-b_1)^{d_1-1}} \cdots + \frac{A_*}{(x-b_1)} + \frac{B_1}{(x-b_2)^{d_2}} + \frac{B_2}{(x-b_2)^{d_2-1}} + \cdots$$

(one term for each expression $(x-b)^k$ dividing denom)

Example: What is the form of the partial fraction expansion of $\frac{x^2+1}{x^2(x+2)}$?

$$\frac{x^2+1}{x^2(x+2)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+2}$$ (blowup at $x=0$, $x=-2$ at $x=0$ blows up like $\frac{1}{x^2}$)

(2) Finding coeff: "Method 1": clear denominators, equate coeff solve system of linear equations.

"Method 2": examine blowup

$$\lim_{x\to 0} \frac{x^2+1}{x+2} = \frac{1}{2}, \qquad \lim_{x\to -2} \frac{x^2+1}{x^2} = \frac{5}{4}$$

so $A = \frac{1}{2}$, $C = 5/4$

Calculate: $$\frac{x^2+1}{x^2(x+2)} - \frac{1}{2x^2} - \frac{5}{4(x+2)} = \frac{4(x^2+1) - 2(x+2) - 5x^2}{4x^2(x+2)} =$$

$$= \frac{-x^2-2x}{4x^2(x+2)} = -\frac{x(x+2)}{4x^2(x+2)} = -\frac{1}{4x}$$

Conclusion: $$\boxed{\frac{x^2+1}{x^2(x+2)} = \frac{1}{2x^2} - \frac{1}{4x} + \frac{5}{4(x+2)}}$$

~~Wel~~ What about $\frac{x^2+1}{x^2(x+2)^2} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{(x+2)^2} + \frac{D}{x+2}$?

Answer: (1) find A, C by taking limit.

(2) subtract $\frac{A}{x^2}$, $\frac{C}{(x+2)^2}$ from $\frac{x^2+1}{x^2(x+2)^2}$

bring to common denom, simplify, cancel.

back to step (0)