31. Manipulating power series (24/3/2017)

Goals:

(1) Substitution in power series
(2) Quiz

Last time: Power series is a series $\sum_{n=0}^{\infty} A_n(x-c)^n$ (centre: c; coefficients: A_n)

* convergence: interval centered at c, radius R.

(1) $R=0$ (only converge at $\{c\}$)

(2) $R=\infty$ (converge all $\mathbb{R}$)

(3) $0<R<\infty$, converge on $(c-R, c+R)$, perhaps also at endpoints

* often, ratio test applies, gives R.

Today: View sum of the series (where convergent) as function of x.

Need to do two things: (1) Go from f to power series expansion
(2) "sum" = find formula for sum of a power series

Today: Manipulating old expansions to get new ones

Example: $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ (for $-1<x<1$)

Math 101 – WORKSHEET 31
MANIPULATING POWER SERIES

1. Manipulating power series: Geometric Series

Recall that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$.

(1) Find a power series representation for

(a) (Final 2014) $\frac{x^3}{1-x}$

Here $\frac{x^3}{1-x} = \frac{1}{1-x} \cdot x^3 = x^3(1 + x + x^2 + x^3 + x^4 + \cdots) = x^3 + x^4 + x^5 + x^6 + \cdots$

$\frac{x^3}{1-x} = x^3 \cdot \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} x^{n+3} = \sum_{m=3}^{\infty} x^m = 0 + 0\cdot x + 0\cdot x^2 + 1\cdot x^3 + 1\cdot x^4 + 1\cdot x^5 + \cdots$

$m = n+3$

(b) (Final 2011) $\frac{1}{1+x^3}$

Let $u = -x^3$. Then $\frac{1}{1+x^3} = \frac{1}{1-u} = \sum_{n=0}^{\infty} u^n = \sum_{n=0}^{\infty} (-x^3)^n = \sum_{n=0}^{\infty} (-1)^n \cdot x^{3n}$.

$= 1 - x^3 + x^6 - x^9 + x^{12} - x^{15} + \cdots$

$= 1 + 0\cdot x + 0\cdot x^2 - 1\cdot x^3 + 0x^4 + 0\cdot x^5 + 1\cdot x^6 + \cdots$

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