Math 101 – SOLUTIONS TO WORKSHEET 4
THE FUNDAMENTAL THEOREM OF CALCULUS

(1) (Differentiating integrals) Evaluate

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\int_0^x e^{t^2}\,\mathrm{d}t$

Solution: By the FTC this is $\boxed{e^{x^2}}$.

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\int_x^1 e^{t^2}\,\mathrm{d}t$

Solution: $\int_x^1 e^{t^2}\,\mathrm{d}t = -\int_1^x e^{t^2}\,\mathrm{d}t$. Applying the FTC we get $\boxed{-e^{x^2}}$.

(c) (Final 2009) $\frac{\mathrm{d}}{\mathrm{d}x}\int_{x^2}^{e^x}\sqrt{\cos t}\,\mathrm{d}t$

Solution: Fix c, and let $F(u) = \int_c^u \sqrt{\cos t}\,\mathrm{d}t$. Then $\int_{x^2}^{e^x}\sqrt{\cos t}\,\mathrm{d}t = \int_c^{e^x}\sqrt{\cos t}\,\mathrm{d}t - \int_c^{x^2}\sqrt{\cos t}\,\mathrm{d}t$ so we need to compute $\frac{\mathrm{d}}{\mathrm{d}x}\left(F(e^x) - F(x^2)\right)$. By the chain rule this is

$$F'(e^x)e^x - F'(x^2)(2x) = \boxed{\sqrt{\cos(e^x)}e^x - 2x\sqrt{\cos(x^2)}\,.}$$

(d) (Final 2014) Let $f(x) = \int_1^x 100(t^2-3t+2)e^{-t^2}\,\mathrm{d}t$. Find the interval(s) on which f is increasing.

Solution: By the FTC, $f'(x) = 100(x^2-3x+2)e^{-x^2} = 100(x-2)(x-1)e^{-x^2}$, which is positive on $\boxed{(-\infty,1)\cup(2,\infty)}$.

(2) Evaluate using anti-derivatives

(a) (Final 2012) $\int_1^2 \frac{x^2+2}{x^2}\,\mathrm{d}x =$

Solution: $\int_1^2 \frac{x^2+2}{x^2}\,\mathrm{d}x = \int_1^2\left(1+\frac{2}{x^2}\right)\mathrm{d}x = \left[x-\frac{2}{x}\right]_{x=1}^{x=2} = (2-1)-(1-2) = \boxed{2}$.

(b) (Final 2007) $\int_{-1}^0 (2x-e^x)\,\mathrm{d}x =$

Solution: $F(x) = x^2 - e^x$ is an anti-derivative, so $\int_{-1}^0 (2x-e^x)\,\mathrm{d}x = \left[x^2-e^x\right]_{x=-1}^{x=0} = 0 - e^0 - \left((-1)^2 - e^{-1}\right) = \boxed{-2+\frac{1}{e}}$.

(c) $\int_3^{10}\left(x^{5/2}+e^{2x}\right)\mathrm{d}x =$

Solution: An anti-derivative is $\frac{2}{7}x^{7/2}+\frac{1}{2}e^{2x}$ so the answer is $\left[\frac{2}{7}x^{7/2}+\frac{1}{2}e^{2x}\right]_{x=3}^{x=10} = \frac{2}{7}10^{7/2}+\frac{1}{2}e^{20}-\frac{2}{7}3^{7/2}-\frac{1}{2}e^6$.

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