Math 412, Spring Term 2014
Midterm Exam

February 24th,2014

Student number:

LAST name:

First name:

Signature:

Instructions

- Do not turn this page over. You will have 50 minutes for the exam (between 11:00–11:50)
- There are 40 points total divided into 4 parts.
- You may not use books, notes or electronic devices of any kind.
- Write in complete English sentences. Proofs should be clear and concise.
- If you are using a result from the textbook, the lectures or the problem sets, state it properly.
- There is an extra blank page at the end of the exam.

1a	/10
1b	/10
2	/10
3	/10
Total	/40

1 A pairing (20 points)

Let $U = \mathbb{R}^3/\mathbb{R}\,(1,1,1)$ and let $V = \left\{\underline{y} \in \mathbb{R}^3 \mid \sum_{i=1}^{3} y_i = 0\right\}$.

a. Show that $\left(\underline{x} + \mathbb{R}\,(1,1,1)\,,\underline{y}\right) = \sum_{i=1}^{3} x_i y_i$ **defines a bilinear form on** $U \times V$ **(10 points)**

b. Show that the form is non-degenerate (10 points)

2 Alternating forms (10 points)

Suppose that $\frac{1}{2} \in F$ and let U be a vector space over F. Construct a natural bijection $\{$alternating bilinear forms on $U\} \leftrightarrow \left(\bigwedge^2 U\right)'$.

3 Problem (10 points)

Let V be a finite-dimensional vector space, and let $T \in \mathrm{End}_F(V)$ be diagonable. Show that the dual map T' is also diagonable.

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