Math 100 – WORKSHEET 4
CONTINUITY: THE IVT; THE DERIVATIVE

1. Continuity

(1) Find c, d, e as appropriate such that each function is continuous on its domain:

$$f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ c & x = 1 \\ d - x^2 & x > 1 \end{cases} \qquad \text{(Final 2013) } g(x) = \begin{cases} ex^2 + 3 & x \ge 1 \\ 2x^3 - e & x < 1 \end{cases}$$

(2) Where are the following functions continuous?

$$f(x) = \frac{1}{\sqrt{7 - x^2}}; \qquad g(x) = \frac{x^2 + 2x + 1}{2 + \cos x}; \qquad h(x) = \frac{2 + \cos x}{x^2 + 2x + 1}$$

(3) (Final 2011) Suppose f, g are continuous such that $g(3) = 2$ and $\lim_{x \to 3} (xf(x) + g(x)) = 1$. Find $f(3)$.

2. The Intermediate Value Theorem

Theorem. *Let $f(x)$ be continuous for $a \le x \le b$. Then $f(x)$ takes every value between $f(a), f(b)$.*

(1) Show that:
   (a) $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \le x \le 1$.

   (b) $\sin x = x + 1$ has a solution.

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(2) (Final 2011) Let $y = f(x)$ be continuous with domain $[0,1]$ and range in $[3,5]$. Show the line $y = 2x+3$ intersects the graph of $y = f(x)$ at least once.

(3) (Final 2015) Show that the equation $2x^2 - 3 + \sin x + \cos x = 0$ has at least two solutions.

3. Definition of the derivative

Definition. $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

(1) Find $f'(a)$ if
  (a) $f(x) = x^2$, $a = 3$.

  (b) $f(x) = \frac{1}{x}$, any a.

  (c) $f(x) = x^3 - 2x$, any a. (you may use $(a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$).

(2) Express the limit as a derivative: $\lim_{h\to 0} \frac{\cos(5+h)-\cos 5}{h}$.