Math 100 – WORKSHEET 9
LOGARITHMS AND LOGARITHMIC DIFFERENTIATION

1. Review of Logarithms

$$\boxed{\log_b(b^x) = b^{\log_b x} = x}$$

$$\boxed{\log_b(xy) = \log_b x + \log_b y} \qquad \boxed{\log_b(x^y) = y \log_b x} \qquad \boxed{\log_b \frac{1}{x} = -\log_b x}$$

(1) $\log\left(e^{10}\right) =$ $\log(2^{100}) =$ (in terms of $\log 2$)

(2) A variant on *Moore's Law* states that computing power doubles every 18 months. Suppose computers today can do N_0 operations per second.

(a) Write a formula predicting the future:

- Computers t years from now will be able to do $N(t)$ operations per second where

$$N(t) =$$

(b) A computing task would take 10 years for today's computers. Suppose we wait 3 years and then start the computation. When will we have the answer?

(c) At what time will computers be powerful enough to complete the task in 6 months?

2. Differentiation

$$\boxed{(\log x)' = \frac{1}{x}}$$

(1) Differentiate

(a) $\frac{\mathrm{d}(\log(ax))}{\mathrm{d}x} =$ $\frac{\mathrm{d}}{\mathrm{d}t} \log\left(t^2 + 3t\right) =$

(b) $\frac{\mathrm{d}}{\mathrm{d}x} x^2 \log(1 + x^2) =$ $\frac{\mathrm{d}}{\mathrm{d}r} \frac{1}{\log(2+\sin r)} =$

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(2) (Logarithmic differentiation) Use $\log(fg) = \log f + \log g$ to differentiate $y = \left(x^2 + 1\right) \cdot \sin x \cdot \frac{1}{\sqrt{x^3+3}} \cdot e^{\cos x}$.

(3) Differentiate using $\boxed{f' = f \times (\log f)'}$

(a) x^x

(b) $(\log x)^{\cos x}$

(c) (Final, 2014) Let $y = x^{\log x}$. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only.