Math 100 – WORKSHEET 11
SCIENTIFIC APPLICATIONS

1. Velocity and acceleration

An object moves by $s = f(t)$. Then the *velocity* is $v(t) = \frac{\mathrm{d}s}{\mathrm{d}t}$ and the *acceleration* is $a(t) = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$.

(1) A particle's position is given by $f(t) = \frac{1}{\pi}\sin(\pi t)$.
   (a) Find the velocity at time t, and specifically at $t = 3$.

   (b) When is the particle moving to the right? to the left?

   (c) When is the particle accelerating? decelerating?

(2) (Final, 2016) An object is thrown straight up into the air at time $t = 0$ seconds. Its height in metres at time t seconds is given by $h(t) = s_0 + v_0 t - 5t^2$. In the first second the object rises by 5 metres. For how many seconds does the object rise before beginning to fall?

(3) A emergency breaking car can decelerate at $9\frac{\mathrm{m}}{\mathrm{s}^2}$. How fast can a car drive so that it can come to a stop within 50m?

Date: 10/10/2019, Worksheet by Lior Silberman.

2. Other applications

(1)

(a) Water is filling a cylindrical container of radius $r = 10\text{cm}$. Suppose that at time t seconds the height of the water is $\left(t + t^2\right)$ cm. How fast is the volume growing?

(b) A rocket is flying in space. The momentum of the rocket is given by the formula $p = mv$, where m is the mass and v is the velocity. At a time where the mass of the rocket is $m = 1000\text{kg}$ and its velocity is $v = 500\frac{\text{m}}{\text{s}}$ the rocket is accelerating at the rate $a = 20\frac{\text{m}}{\text{s}^2}$ and losing mass at the rate $10\frac{\text{kg}}{\text{s}}$. Find the rate of change of the momentum with time.

(2) A ball is falling from rest in air. Its height at time t is given by

$$h(t) = H_0 - gt_0\left(t + t_0 e^{-t/t_0} - t_0\right)$$

where H_0 is the initial height and t_0 is a constant.

(a) Find the velocity of the ball. $v(t) =$

(b) Find the acceleration. $a(t) =$

(c) Find $\lim_{t\to\infty} v(t)$