Math 100 – WORKSHEET 18
THE MVT AND CURVE SKETCHING

1. Applying the MVT

Theorem. *Let f be defined and continuous on $[a, b]$, differentiable on (a, b). Then there is c between a, b such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.*
Equivalently, for any x there is c between a, x so that $f(x) = f(a) + f'(c)(x - a)$.

(1) Suppose $f'(x) = \frac{e^x}{x+\pi}$ for $0 \le x \le 2$. Give an upper bound for $f(2) - f(0)$.

(2) (Final, 2015) Show that $2x^2 - 3 + \sin x + \cos x = 0$ has at most two solutions.

(3) Suppose f satisfies the hypotheses of the MVT and that $f'(x) > 0$ for all $x \in (a, b)$. Show that $\frac{f(b)-f(a)}{b-a} > 0$, and hence that $f(b) > f(a)$.

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2. The shape of a the graph

(4) Let f be twice differentiable on $[a,b]$.

(a) Suppose first that $f(a)=f(b)=0$ and that f is positive somewhere between a,b. Show that there is c between a,b so that $f''(c)<0$.

(b) Now let $f(a),f(b)$ take any values, but suppose $f''(x)>0$ on (a,b). Let $L: y=mx+n$ be the line through $(a,f(a)),(b,f(b))$. Applying part (a) to $g(x)=f(x)-(mx+n)$ show that the graph of f lies below the line L.

Definition. We say f is *concave up* (or "convex") on an interval $[a,b]$ if its graph lies under the secant lines in this interval. This is true, for example, if $f''>0$ on (a,b). We say f is *concave down* (or "concave") on the interval if its graph lies below the scant lines, in particular when $f''<0$ on (a,b). We say that f has an *inflection point* at x_0 if its second derivative changes sign there.