Lior Silberman's Math 312: ComPAIR Assignment 5

- This assignment is due Wednesday, 7/4/2021 at noon (Vancover time)
- Comparisons are due Sunday, 11/4/2021 at 11pm (Vancouver time).

Recall that for a modulus m each integer a is congruent to a unique *reduced residue* mod m (an integer in the range $[0, m-1]$ and if m is odd also to a unique *balanced residue* (an integer in the range $\left[-\frac{m-1}{2}, \frac{m-1}{2}\right]$ (if m is even we can use the range $\left[-\frac{m}{2}+1, \frac{m}{2}\right]$ or $\left[-\frac{m}{2}, \frac{m}{2}-1\right]$.

1. Let p be an odd prime.
   (a) Give a formula for s depending on p.

$$\begin{aligned} s &= \#\left\{1 \le t \le \frac{p-1}{2} \mid \text{the reduced residue of } -t \text{ is between } \left[\frac{p+1}{2}, p-1\right]\right\} \\ &= \#\left\{1 \le t \le \frac{p-1}{2} \mid \text{the balanced residue of } -t \text{ is between } \left[-\frac{p-1}{2}, -1\right]\right\} \end{aligned}$$

   (b) Use Gauss's Lemma to conclude that $\left(\frac{-1}{p}\right) = \begin{cases} +1 & p \equiv 1\,(4) \\ -1 & p \equiv 3\,(4) \end{cases}$.

2. Let p be an odd prime.
   (a) Give a formula for s depending on p.

$$\begin{aligned} s &= \#\left\{1 \le t \le \frac{p-1}{2} \mid \text{the reduced residue of } 2t \text{ is between } \left[\frac{p+1}{2}, p-1\right]\right\} \\ &= \#\left\{1 \le t \le \frac{p-1}{2} \mid \text{the balanced residue of } 2t \text{ is between } \left[-\frac{p-1}{2}, -1\right]\right\} \end{aligned}$$

   (b) Use Gauss's Lemma to conclude that $\left(\frac{2}{p}\right) = \begin{cases} +1 & p \equiv \pm 1\,(8) \\ -1 & p \equiv \pm 3\,(8) \end{cases}$

For parts (a), the key ideas are (1) *edge cases* for t (for each range of consecutive t where the claim holds, what are the endpoints? and (2) In part 2(a) *division into cases* for p: the formula for the edge case might depend on the class of p mod 8.