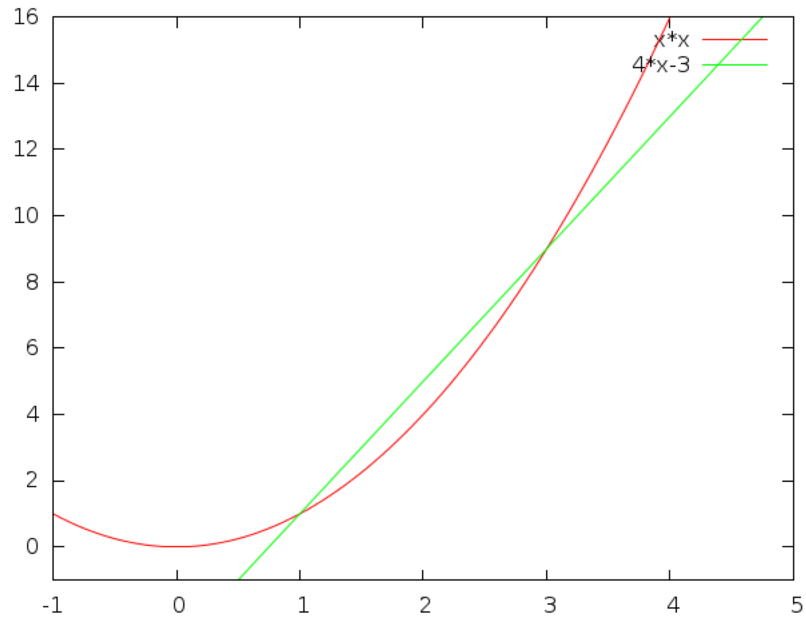
Math 100 – SOLUTIONS TO WORKSHEET 1
LIMITS

1. The slope of a graph



(1) Find the slope of the line through points $P(1,1)$ and $Q(x,x^2)$ where:
 (a) $x=3$
 Solution: $Q=(3,9)$ so slope is $\frac{\Delta y}{\Delta x}=\frac{9-1}{3-1}=4$
 (b) $x=1.1$
 Solution: $Q=(1.1,1.21)$ so slope is $\frac{\Delta y}{\Delta x}=\frac{1.21-1}{1.1-1}=\frac{0.21}{0.1}=2.1$
 (c) $x=1.01$
 Solution: $Q=(1.01,1.0201)$ so slope is $\frac{\Delta y}{\Delta x}=\frac{1.0201-1}{1.01-1}=\frac{0.0201}{0.01}=2.01$
 (d) $x=1.001$
 Solution: $Q=(1.001,1.002001)$ so slope is $\frac{\Delta y}{\Delta x}=\frac{1.002001-1}{1.001-1}=\frac{0.002001}{0.001}=2.001$

What is the slope of the line tangent to the curve at $P(1,1)$? What is its equation?
Solution: The slope is 2, so the line is $y-1=2(x-1)$ or $y=2x-1$.

2. Limits

(2) Evaluate $f(x)=\frac{x-3}{x^2-x-6}$ at $x=2.9$, 2.99, 2.999, 3.1,3.01, 3.001. What is $\lim_{x\to3}f(x)$?

Solution: For $x\neq3$ we have $\frac{x-3}{x^2-x-6}=\frac{x-3}{(x-3)(x+2)}=\frac{1}{x+2}$ so $\lim_{x\to3}f(x)=\lim_{x\to3}\frac{1}{x+2}=\boxed{\frac{1}{5}}$

(3) Evaluate
 (a) $\lim_{x\to1}\sin(\pi x)$
 Solution: The function is nice and $\lim_{x\to1}\sin(\pi x)=\sin(\pi)=0$.

Date: 9/9/2021, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

(b) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2}$.

Solution: $\frac{e^x(x-1)}{x^2+x-2} = \frac{e^x(x-1)}{(x-1)(x+2)} = \frac{e^x}{x+2} \xrightarrow[x\to 1]{} \frac{e^1}{1+2} = \boxed{\frac{e}{3}}$.

(c) $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1+x}}{3x}$

Solution: We have

$$\begin{aligned}
\frac{\sqrt{1+2x}-\sqrt{1+x}}{3x} &= \frac{\sqrt{1+2x}-\sqrt{1+x}}{3x} \cdot \frac{\sqrt{1+2x}+\sqrt{1+x}}{\sqrt{1+2x}+\sqrt{1+x}} \\
&= \frac{\left(\sqrt{1+2x}-\sqrt{1+x}\right)\left(\sqrt{1+2x}+\sqrt{1+x}\right)}{3x\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \\
&= \frac{(1+2x)-(1+x)}{3x\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \\
&= \frac{x}{3x} \cdot \frac{1}{\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \\
&= \frac{1}{3\left(\sqrt{1+2x}+\sqrt{1+x}\right)} \xrightarrow[x\to 0]{} \frac{1}{3\left(\sqrt{1}+\sqrt{1}\right)} = \boxed{\frac{1}{6}}.
\end{aligned}$$

(4) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

(a) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 3 & x = 1 \\ 2-x^2 & x > 1 \end{cases}$.

Solution: From the left $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{x} = \sqrt{1} = 1$. From the right $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2-x^2 = 2-1 = 1$ so the limit exists and equals 1. Note that the value at $x = 1$ doesn't enter the picture.

(b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 4-x^2 & x > 1 \end{cases}$.

Solution: From the left $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{x} = \sqrt{1} = 1$. From the right $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 4-x^2 = 4-1 = 3$ so the limit does not exist.