

Math 100 – WORKSHEET 4
CONTINUITY; THE IVT

1. CONTINUITY

(1) Which of these functions are continuous everywhere? Why?

(a) $f(x) = \begin{cases} x & x < 0 \\ \cos x & x \geq 0 \end{cases}$

(b) $f(x) = \begin{cases} x & x < 0 \\ \sin x & x \geq 0 \end{cases}$

(2) Let $f(x) = \frac{x^3 - x^2}{x - 1}$.

(a) Why is $f(x)$ discontinuous at $x = 1$?

(b) Find b such that $g(x) = \begin{cases} f(x) & x \neq 1 \\ b & x = 1 \end{cases}$ is continuous everywhere.

(c) Find c, d such that $h(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ c & x = 1 \\ d - x^2 & x > 1 \end{cases}$ is continuous.

(d) (Final 2013) For which value of the constant c is $f(x) = \begin{cases} cx^2 + 3 & x \geq 1 \\ 2x^3 - c & x < 1 \end{cases}$ continuous on $(-\infty, \infty)$?

(3) Where are the following functions continuous?

$$f(x) = \frac{1}{\sqrt{7 - x^2}}; \quad g(x) = \frac{x^2 + 2x + 1}{2 + \cos x}; \quad h(x) = \frac{2 + \cos x}{x^2 + 2x + 1}; \quad k(x) = \log(\sin x)$$

- (4) (Final 2011) Suppose f, g are continuous such that $g(3) = 2$ and $\lim_{x \rightarrow 3} (xf(x) + g(x)) = 1$. Find $f(3)$.

2. THE INTERMEDIATE VALUE THEOREM

Theorem. *Let $f(x)$ be continuous for $a \leq x \leq b$. Then $f(x)$ takes every value between $f(a), f(b)$.*

- (5) Show that $f(x) = 2x^3 - 5x + 1$ has a zero in $0 \leq x \leq 1$.

- (6) (Final 2011) Let $y = f(x)$ be continuous with domain $[0, 1]$ and range in $[3, 5]$. Show the line $y = 2x + 3$ intersects the graph of $y = f(x)$ at least once.

- (7) (Final 2010) Two points on Earth are called *antipodal* if they are exactly opposite to each other. Show that, at any given moment, there are two antipodal points on the equator with exactly the same temperature.