Math 100 – WORKSHEET 15
TAYLOR REMAINDER ESTIMATES

1. Review: Taylor expansion

Let $c_k = \frac{f^{(k)}(a)}{k!}$. The nth order Taylor expansion of $f(x)$ about $x = a$ is the polynomial
$$T_n(x) = c_0 + c_1(x-a) + \cdots + c_n(x-a)^n$$

(1) Estimate $(4.1)^{3/2}$ using a linear and a quadratic approximation.

(2) The third-order expansion of $h(x)$ about $x = 2$ is $3 + \frac{1}{2}(x-2) + 2(x-2)^3$. What are $h'(2)$ and $h''(2)$?

(3) (Final, 2016) Find the 3rd order Taylor expansion of $(x+1)\sin x$ about $x = 0$.

2. Error estimate 1

Let $R_1(x) = f(x) - T_1(x)$ be the *remainder*. Then there is c between a and x such that
$$R_1(x) = \frac{f^{(2)}(c)}{2!}(x-a)^2$$

(4) Estimate the error in the linear approximations to $(4.1)^{3/2}$.

(5) (Final, 2012) Show $-\frac{5}{32} \le \log\left(\frac{8}{9}\right) \le -\frac{1}{9}$ using the linear approximation to $f(x) = \log\left(1 - x^2\right)$.

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3. Higher order error estimates

Let $R_n(x) = f(x) - T_n(x)$ be the *remainder*. Then there is c between a and x such that

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

(6) Estimate the magnitude of the error in the quadratic approximation to $(4.1)^{3/2}$.

(7) (Quiz, 2015) Consider a function f such that $f^{(4)}(x) = \frac{\cos(x^2)}{3-x}$. Show that, when approximating $f(0.5)$ using its third-degree MacLaurin polynomial, the absolute value of the error is less than $\frac{1}{500}$.

(8) (Final, 2012) Show that for all $-1 \le x \le 1$ we have

$$0 \le \cos(x) - \left(1 - \frac{x^2}{2}\right) \le \frac{1}{24}\,.$$