

**Math 100 – WORKSHEET 17**  
**OPTIMIZATION**

Problem-solving steps: (0) read carefully, draw picture; (1) fix coordinate system, name variables; (2) enforce relations; (3) create objective function; (4) calculus; (5) endgame.

- (1) (Final 2012) The right-angled triangle  $\triangle ABP$  has the vertex  $A = (-1, 0)$ , a vertex  $P$  on the semicircle  $y = \sqrt{1 - x^2}$ , and another vertex  $B$  on the  $x$ -axis with the right angle at  $B$ . What is the largest possible area of this triangle?

- (2) (Final 2010) A river running east-west is 6km wide. City A is located on the shore of the river; city B is located 8km to the east on the opposite bank. It costs \$40/km to build a bridge across the river, \$20/km to build a road along it. What is the cheapest way to construct a path between the cities?

- (3) (Final 2019) Among all rectangles inscribed in a given circle, which one has the largest perimeter? Prove your answer.