

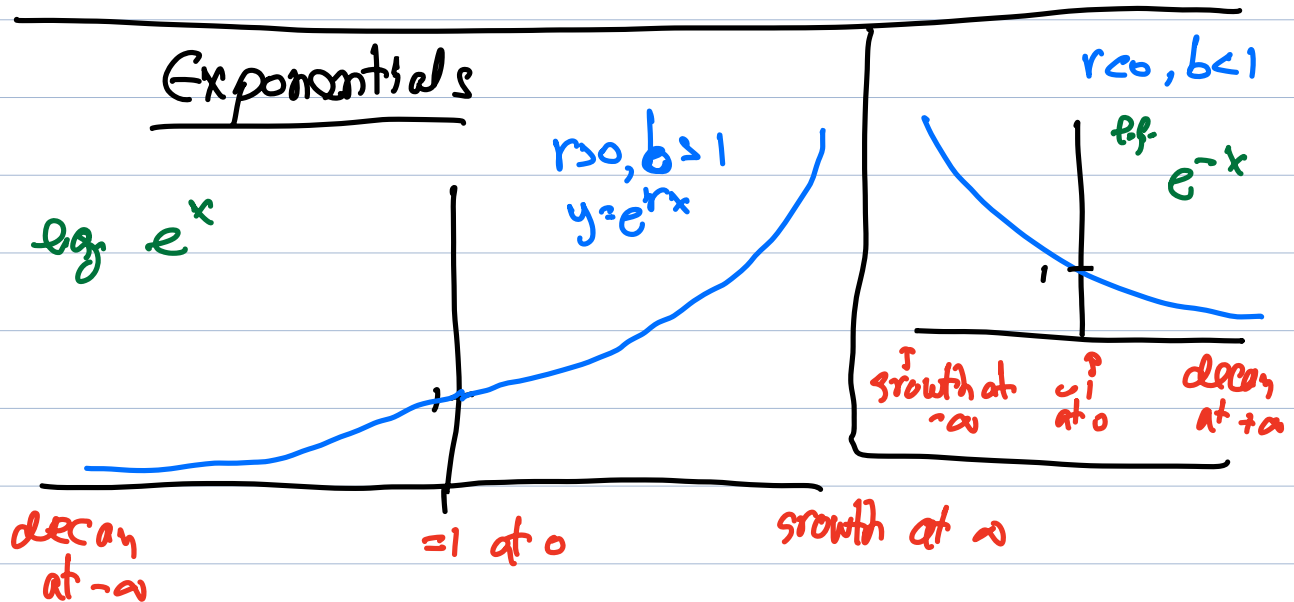
Math 100, lecture 1 Asymptotics

Today: (1) Intro (2) The ladder of functions
(3) Asymptotics of expressions.

Taxonomy: A **power law** is a function $x \mapsto C \cdot x^n$
(n called **index**)

An **exponential** is a function $x \mapsto C \cdot e^{rx}$
 $x \mapsto C \cdot b^x$ \downarrow $b=e^r$

(r called **exponential rate**)
 b called **base**)

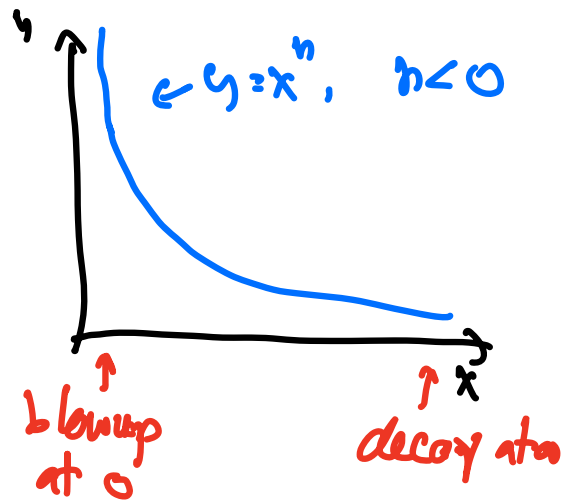
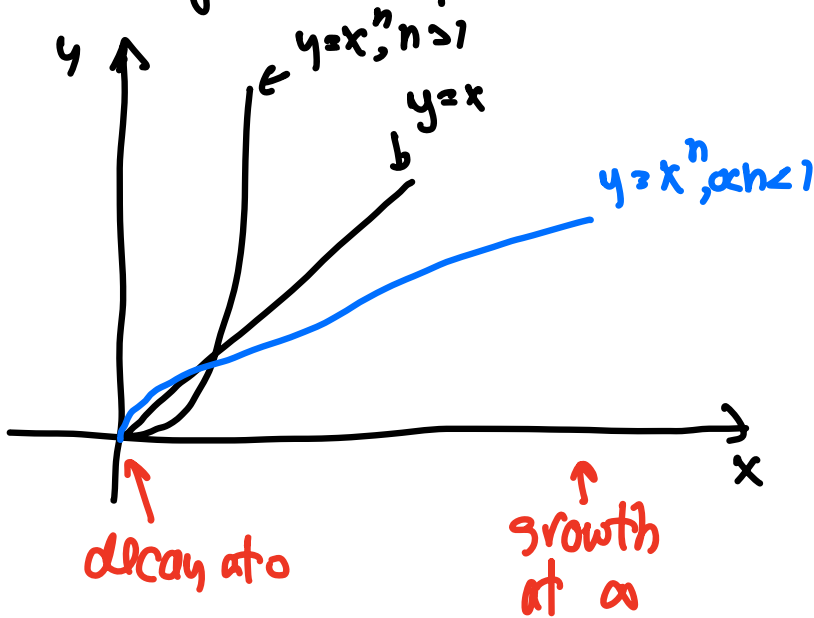


1. THE LADDER OF FUNCTIONS

(1) Classify the following functions into *power laws* / *power functions* and *exponentials*: x^3 , πx^{102} , e^{2x} , $c\sqrt{x}$, $-\frac{8}{x}$, 7^x , $8 \cdot 2^x$, $-\frac{1}{\sqrt{3}} \cdot \frac{1}{2^x}$, $\frac{9}{x^{7/2}}$, x^e , π^x , $\frac{A}{x^b}$.

Power laws: x^3 , $\pi \cdot x^{102}$, $C\sqrt{x} = C \cdot x^{1/2}$, $-\frac{8}{x} = -8 \cdot x^{-1}$, $9x^{-7/2}$, x^e , Ax^{-b}
Exponentials: e^{2x} , $8 \cdot 2^x$, $-\frac{1}{\sqrt{3}} \cdot 2^{-x}$, π^x

Graphs of power laws and exponentials



Fact: exponentials eventually beat power laws.

(2) Order the following functions from small to large asymptotically as $x \rightarrow \infty$:

(a) $1, \sqrt{x}, x^{-1/2}, x^{1/3}, e^x, x^{-1/3}, 10^6 x^{2024}, e^{-x}, e^{x^2}, \frac{2024}{x^{100}}, 5^x, x$.

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$\ll 10^6 x^{2024} \ll e^x \ll 5^x \ll e^{x^2}$

(b) Extra: add in $\log x, e^{\sqrt{x}}, (\log x)^2, \log \log x, \frac{1}{\log x}$.

(c) Repeat (a), this time as $x \rightarrow 0^+$.