

Math 100:V02 – WORKSHEET 2
LIMITS

1. ASYMPTOTICS

- (1) How does the each expression behave when x is large? small? what is x is large but negative? Sketch a plot

(a) $ax^3 - bx^5$ ($a, b > 0$)

(b) $e^x - x^4$

- (2) Say each expression in words, and then determine its asymptotics near 0 and near ∞ .

(a) $e^{|x-5|^3}$

(b) $\frac{1+x}{1+2x-x^2}$

(c) $\frac{e^x + A \sin x}{e^x - x^2}$

(d) $\frac{Ae^{rt} + Be^{-st}}{t+t^2}$ where $r, s > 0$ and $A, B \neq 0$.

(3) Find the asymptotics of the indicated expression at the given point.

(a) $\frac{x^5 + Ax^3 + x}{Bx^4 - x^2}$ as $x \rightarrow 0$.

(b) $\frac{x^2 + 1}{x - 4}$ as $x \rightarrow 3$.

(c) $f(x) = \frac{x^2 + 1}{x - 4}$ as $x \rightarrow 4$.

(d) $f(x) = x^2 - 1$ as $x \rightarrow 1$.

2. LIMITS

(4) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.

(a) $\lim_{x \rightarrow 5} (x^3 - x)$

(b) $\lim_{x \rightarrow 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 3 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

(c) $\lim_{x \rightarrow 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \leq x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.

(5) Let $f(x) = \frac{x-3}{x^2+x-12}$.

(a) (Final 2014) What is $\lim_{x \rightarrow 3} f(x)$?

(b) What about $\lim_{x \rightarrow -4} f(x)$?

(6) Evaluate

(a) $\lim_{x \rightarrow \infty} \frac{e^x + A \sin x}{e^x - x^2}$

(b) $\lim_{x \rightarrow 0} \frac{e^x + A \sin x}{e^x - x^2}$

(c) $\lim_{x \rightarrow -\infty} \frac{e^x + A \sin x}{e^x - x^2}$

(7) Evaluate

(a) $\lim_{x \rightarrow 2} \frac{x+1}{4x^2-1}$

(b) (Final, 2014) $\lim_{x \rightarrow -3^+} \frac{x+2}{x+3}$.

(c) $\lim_{x \rightarrow 1} \frac{e^x(x-1)}{x^2+x-2}$

(d) $\lim_{x \rightarrow -2^-} \frac{e^x(x-1)}{x^2+x-2}$

(e) $\lim_{x \rightarrow 1} \frac{1}{(x-1)^2}$

(f) $\lim_{x \rightarrow 4} \frac{\sin x}{|x-4|}$

(g) $\lim_{x \rightarrow \frac{\pi}{2}^+} \tan x$, $\lim_{x \rightarrow \frac{\pi}{2}^-} \tan x$.

3. LIMITS AT INFINITY

(6) Evaluate

(a) $\lim_{x \rightarrow \infty} \frac{x^2+1}{x-3}$

(b) (Final, 2015) $\lim_{x \rightarrow -\infty} \frac{x+1}{x^2+2x-8}$

(c) (Quiz, 2015) $\lim_{x \rightarrow -\infty} \frac{3x}{\sqrt{4x^2+x}-2x}$