

**Math 100:V02 – WORKSHEET 3**  
**THE DERIVATIVE**

1. THREE VIEWS OF THE DERIVATIVE

- (1) Let  $f(x) = x^2$ , and let  $a = 2$ . Then  $(2, 4)$  is a point on the graph of  $y = f(x)$ .
- (a) Let  $(x, x^2)$  be another point on the graph, close to  $(2, 4)$ . What is the slope of the line connecting the two? What is the limit of the slopes as  $x \rightarrow 2$ ?
- (b) Let  $h$  be a small quantity. What is the asymptotic behaviour of  $f(2 + h)$  as  $h \rightarrow 0$ ? What about  $f(2 + h) - f(2)$ ?
- (c) What is  $\lim_{h \rightarrow 0} \frac{(2+h)^2 - 2^2}{h}$ ?
- (d) What is the equation of the line tangent to the graph of  $y = f(x)$  at  $(2, 4)$ ?
- (2) **★** An enzymatic reaction occurs at rate  $k(T) = T(40 - T) + 10T$  where  $T$  is the temperature in degrees celsius. The current temperature of the solution is  $20^\circ\text{C}$ . Should we increase or decrease the temperature to increase the reaction rate?

## 2. DEFINITION OF THE DERIVATIVE

**Definition.**  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  or  $f(a+h) \approx f(a) + f'(a)h$

(3) Find  $f'(a)$  if

(a)  $\star f(x) = x^2, a = 3.$

(b)  $\star\star f(x) = \frac{1}{x},$  any  $a.$

(c)  $\star\star f(x) = x^3 - 2x,$  any  $a$  (you may use  $(a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$ ).

(4)  $\star\star$  Express the limits as derivatives:  $\lim_{h \rightarrow 0} \frac{\cos(5+h) - \cos 5}{h}, \lim_{x \rightarrow 0} \frac{\sin x}{x}$

(5)  $\star\star\star$  (Final, 2015, variant – gluing derivatives) Is the function

$$f(x) = \begin{cases} x^2 & x \leq 0 \\ x^2 \cos \frac{1}{x} & x > 0 \end{cases}$$

differentiable at  $x = 0$ ?

**Fact.** The derivative of  $x^n$  with respect to  $x$  is  $nx^{n-1}$ .

### 3. THE TANGENT LINE

**Definition.** The line tangent to the graph  $y = f(x)$  at  $x = a$  is the line  $y = f'(a)(x - a) + f(a)$

(6) ★ (Final, 2015) Find the equation of the line tangent to the function  $f(x) = \sqrt{x}$  at  $(4, 2)$ .

(7) ★★(Final 2015) The line  $y = 4x + 2$  is tangent at  $x = 1$  to which function:  $x^3 + 2x^2 + 3x$ ,  $x^2 + 3x + 2$ ,  $2\sqrt{x + 3} + 2$ ,  $x^3 + x^2 - x$ ,  $x^3 + x + 2$ , none of the above?

(8) ★★★ Find the lines of slope 3 tangent to the curve  $y = x^3 + 4x^2 - 8x + 3$ .

(9) ★★★ The line  $y = 5x + B$  is tangent to the curve  $y = x^3 + 2x$ . What is  $B$ ?

4. LINEAR APPROXIMATION

**Definition.**  $f(a + h) \approx f(a) + f'(a)h$

(10) Estimate

(a)  $\star \sqrt{1.2}$

(b)  $\star$  (Final, 2015)  $\sqrt{8}$

(c)  $\star$  (Final, 2016)  $(26)^{1/3}$