Math 100:V02 – WORKSHEET 4
CALCULATING DERIVATIVES

1. Definition of the derivative

Definition. $f(a+h) \approx f(a) + f'(a)h$ (or $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$)

(1) Find $f'(a)$ if
  (a) $f(x) = x^2$, $a = 3$.

  (b) $f(x) = \frac{1}{x}$, any a.

  (a) $f(x) = x^3 - 2x$, any a (you may use $(a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$).

(2) Express the limits as derivatives: $\lim_{h\to 0} \frac{\cos(5+h)-\cos 5}{h}$, $\lim_{x\to 0} \frac{\sin x}{x}$

(3) (Final, 2015, variant – gluing derivatives) Is the function

$$f(x) = \begin{cases} x^2 & x \le 0 \\ x^2 \cos \frac{1}{x} & x > 0 \end{cases}$$

differentiable at $x = 0$?

Date: 23/1/2024, Worksheet by Lior Silberman.

2. The tangent line

Definition. The *line tangent to the graph $y = f(x)$ at $x = a$* is the line $y = f'(a)(x-a) + f(a)$

(4) (Final, 2015) Find the equation of the line tangent to the function $f(x) = \sqrt{x}$ at $(4, 2)$.

(5) (Final 2015) The line $y = 4x + 2$ is tangent at $x = 1$ to which function: $x^3 + 2x^2 + 3x$, $x^2 + 3x + 2$, $2\sqrt{x+3} + 2$, $x^3 + x^2 - x$, $x^3 + x + 2$, none of the above?

(6) Find the lines of slope 3 tangent to the curve $y = x^3 + 4x^2 - 8x + 3$.

(7) The line $y = 5x + B$ is tangent to the curve $y = x^3 + 2x$. What is B?

3. Linear approximation

Definition. $f(a+h) \approx f(a) + f'(a)h$

(8) Estimate

(a) $\star$ $\sqrt{1.2}$

(b) $\star$ (Final, 2015) $\sqrt{8}$

(c) $\star$ (Final, 2016) $(26)^{1/3}$

4. Arithmetic of derivatives

Fact. $(af+bg)' = af' + bg'$, $(fg)' = f'g + fg'$, $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
$\frac{d}{dx}x^n = nx^{n-1}$.

(2) Differentiate

(a) $\star$ $f(x) = 6x^\pi + 2x^e - x^{7/2}$

(b) $\star$ (Final, 2016) $g(x) = x^2 e^x$ (and then also $x^a e^x$)

(c) $\star$ (Final, 2016) $h(x) = \frac{x^2+3}{2x-1}$

(d) $\star$ $\frac{x^2+A}{\sqrt{x}}$

(3) $\star$ Let $f(x) = \frac{x}{\sqrt{x+A}}$. Given that $f'(4) = \frac{3}{16}$, give a quadratic equation for A.

(4) Suppose that $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$, $g'(1) = 4$.

(a) $\star$ What are the linear approximations to f and g at $x = 1$? Use them to find the linear approximation to fg at $x = 1$.

(b) $\star$ Find $(fg)'(1)$ and $\left(\frac{f}{g}\right)'(1)$.

(5) Evaluate

(a) $\star$ $(x \cdot x)'$ and $(x') \cdot (x')$. What did we learn?

(b) $\star$ $\left(\frac{x}{x}\right)'$ and $\frac{(x')}{(x')}$. What did we learn?