6. INVERSE TRIG (9/10/2024)

Notes.

(1) WW5 deadline has been extended due to midterm.

(2) Group Project 2 is live.

Goals.

(1) Inverse trig functions

(2) (Differentiation: upcoming small class)

Last Time. **Chain rule.**

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

Application: (log. diff) $(\log f)' = \frac{1}{f} \cdot f' \implies f' = f \cdot (\log f)'$

Diff along curves: if $y = y(x)$ can diff $f(x,y)$.

Inverse functions

Say $y = f(x)$, The *inverse function* answers the question:

"given y what is x?"

Examples x ranges over students, $f(x) = $ student number.

$\quad\quad\quad f^{-1}(\text{student number}) = \text{student}$ with that number.

Example,   look at $y = x^2$!
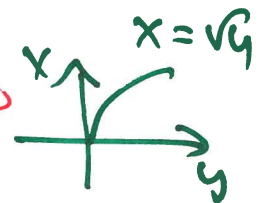
The equation $x^2 = y$ has 2 solutions if $y > 0$.

$\quad\quad\quad\quad\quad\quad\quad\quad\quad 0 \quad\quad " \quad\quad$ if $\quad y < 0$

$\Rightarrow$ to define inverse need to choose one of the solutions,

$\sqrt{}$ will be defined on $\{y \geq 0\}$ $\leftarrow$ range of x^2.

restrict x^2 to $[0, \infty)$: $\quad$ not $\quad\quad$ $x = \sqrt{y}$

Conclusions

Given f, ① restrict domain so that f takes every value once.

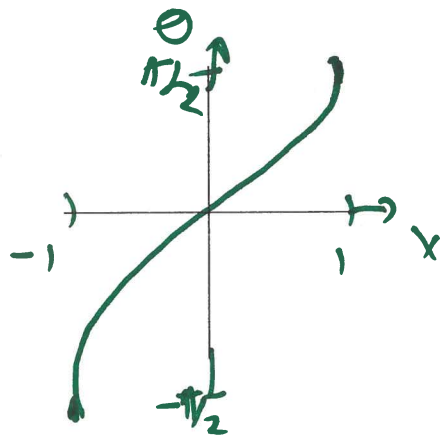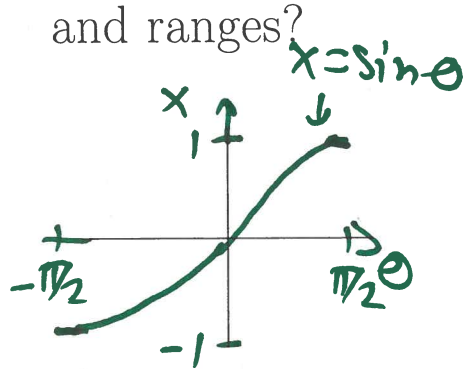② ~~define~~ f^{-1} for each y in range of f define $f^{-1}(y)$ to be the x so that $f(x) = y$.

Examples: ① $\sqrt{y}$ inverse to $y = x^2$ on $[0, \infty)$ $\quad \left(\begin{array}{c}\text{domain} \\ \text{is } [0, \infty)\end{array}\right)$

② $\log y$ inverse to e^x $\quad \left(\begin{array}{c}\text{domain} \\ \text{is } (0, \infty)\end{array}\right)$
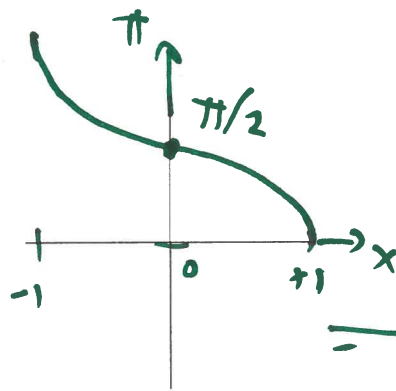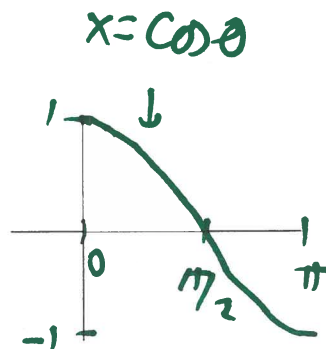
2. INVERSE TRIG FUNCTIONS

(7) Draw on the following axes graphs of $\sin\theta$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\cos\theta$ on $[0, \pi]$ and $\tan\theta$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then of their inverse functions. What are their domains and ranges?

$x = \sin\theta$

$x = \cos\theta$

$x = \tan\theta$
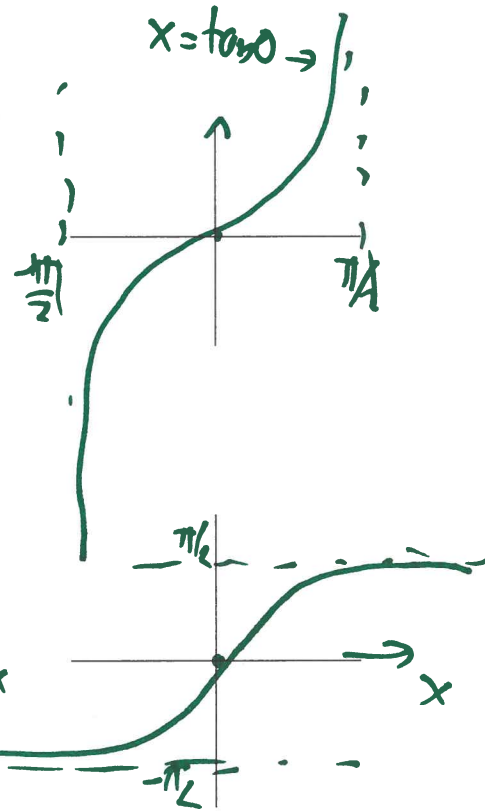
$\theta = \arcsin(x)$

$\arcsin\left(\frac{1}{2}\right) =$

$\theta = \arccos(x)$ — domain $[-1, 1]$

$\theta = \arctan(x)$
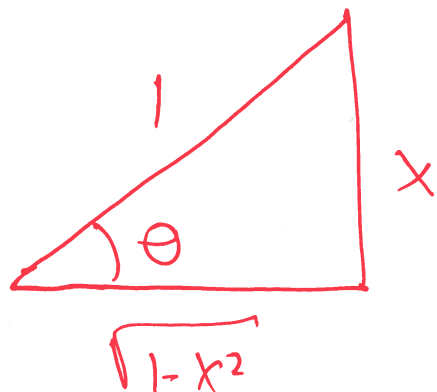domain $(-\infty, \infty)$

$\lim_{x \to \infty} \arctan x = \frac{\pi}{2}$

$\lim_{x \to -\infty} \arctan x = -\frac{\pi}{2}$

Idea

Given x

$\theta = \arcsin x$

$\Downarrow$

$\sin \theta = x$

fill one side x
one side 1
Compute third side
from Pythagoras