

MATH 100A: 1A1 lecture 12, 20/11/2024

Newton's Method

no more
midterms/
group
projects

(continued in SC12)

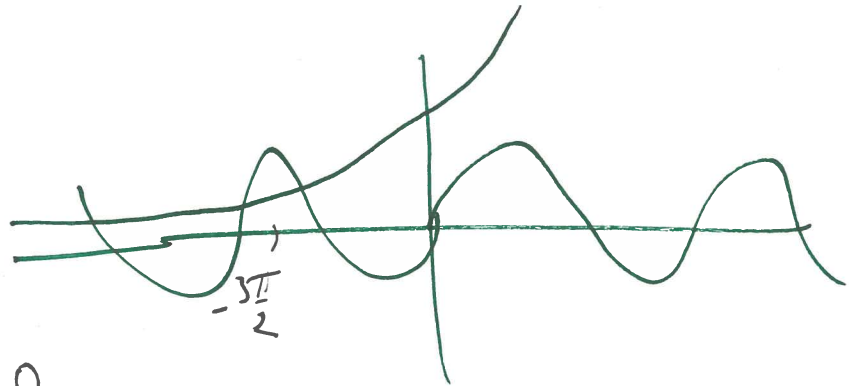
Problem: can't solve equations, but we need to.

Examples:

$$x^2 = 6$$

$$x^3 + x + 9 = 0$$

$$e^x = \sin x$$



Simple method: $e^0 = 1 > \sin 0$

$$e^{-3\pi/2} < 1 = \sin(-2\pi \rightarrow \frac{\pi}{2})$$

halfway is $-3\pi/4$

let $f(x) = e^x - \sin x$: $f(0) > 0$, $f(-\frac{3\pi}{2}) < 0$

then iterate

$f(-3\pi/4)$ will make an interval
with crossing

key idea ①: iteration

Successive approximation

Idea ②: linear approx.

- ① given guess, make linear approx
- ② solve linear equation to find next guess
- ③ repeat.

Example $f(x) = x^3 + x + 9$ want: x s.t. $f(x) = 0$

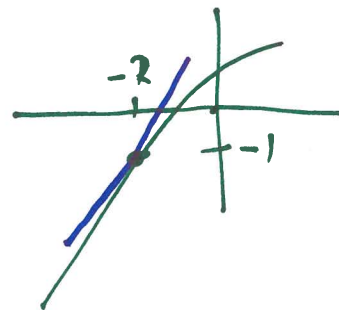
see $f(0) = 9$, $f(-1) = 7$, $f(-2) = -1$, $f(-3) = -2$.

(use asymptotics, plugging in values to get initial guess)

① linear approx to f at $x_1 = -2$?

$$f'(x) = 3x_1^2 + 1, \quad f'(-2) = 13$$

$$f(x) \approx -1 + 13(x+2)$$



② linear approx crosses axis at: $-1 + 13(x_2 + 2) = 0$

$$\text{so } 13x_2 = -25, \quad x_2 = -\frac{25}{13}$$

Do it again: $f'(x_2) = 3 \cdot \frac{625}{169} + 1$

\Rightarrow linear approx \rightarrow guess x_3

General function

Start with guess a . linear approx: $f(x) \approx f(a) + f'(a)(x-a)$

to find next guess solve: $f(a) + f'(a)(x-a) = 0$

get: $x = a - \frac{f(a)}{f'(a)}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example: if $f(x) = x^2 - b$

formula is $x_{n+1} = \frac{x_n + b/x_n}{2}$

(known to
Greeks)