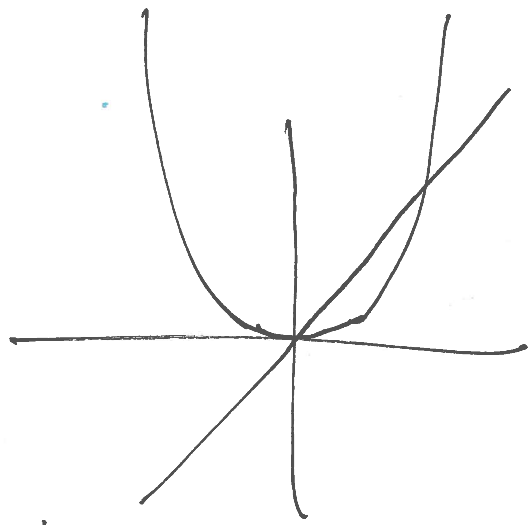
<u>Asymptotic comparison</u>

As $x \to \infty$, x^2 will be much bigger than x.

<u>Write:</u> $x^2 >> x$ <u>Say</u>: x^2 asymptotically dominates x

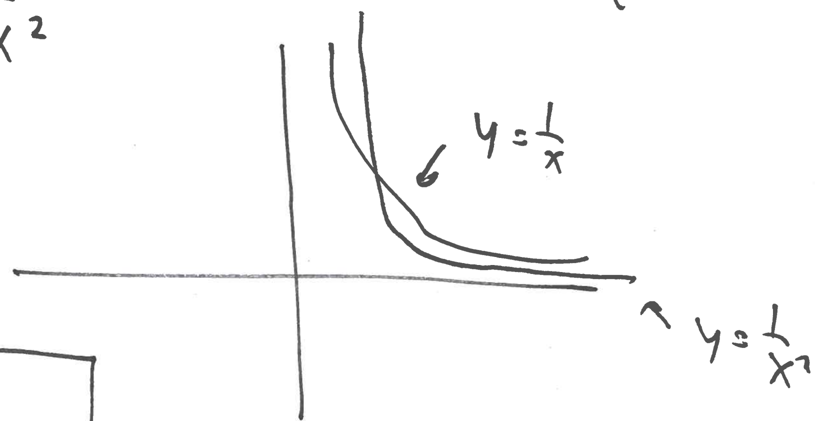
Also $\frac{1}{10^6} x^2 >> 10^6 x$

~~[illegible]~~ As $x \to 0$, $x >> x^2$.



As $x \to \infty$, $\frac{1}{x} >> \frac{1}{x^2}$

As $x \to 0$ $\frac{1}{x} << \frac{1}{x^2}$



Exponentials get faster
than power laws

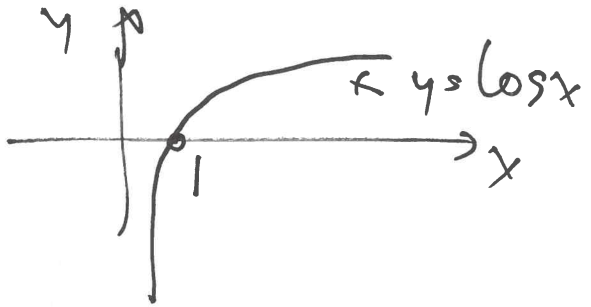
(2) Order the following functions from small to large *asymptotically as* $x \to \infty$:

(a) 1, $\sqrt{x}$, $x^{-1/2}$, $x^{1/3}$, e^x, $x^{-1/3}$, $10^6 x^{2024}$, e^{-x}, e^{x^2}, $\frac{2024}{x^{100}}$, 5^x, x.

$$e^{-x} \ll \frac{2024}{x^{100}} \ll x^{-\frac{1}{2}} \ll x^{-\frac{1}{3}} \ll 1 \ll x^{1/3} \ll x^{1/2} \ll x \ll$$

$$\ll 10^6 x^{2024} \ll e^x \ll 5^x \ll e^{x^2}$$

$$\left(e^{x^2} = (e^x)^x\right)$$



$\boxed{\begin{array}{l}\underline{\text{Defi:}}\\ \log x = \log_e x\end{array}}$

(b) Extra: add in $\log x$, $e^{\sqrt{x}}$, $(\log x)^2$, $\log\log x$, $\frac{1}{\log x}$.

① $\log x$ grows / blows up <u>slower</u> than any power law

(3) How does the each expression behave when x is large? small? what is x is large but negative? Sketch a plot

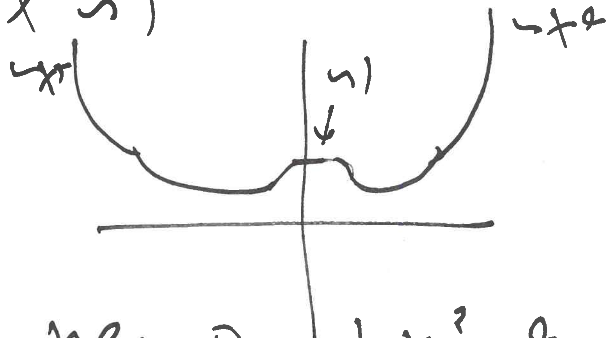
(a) $1 - x^2 + x^4$ ("Mexican hat potential")

is asymptotic to

As $x \to \infty$, x^4 dominates $x^2, 1$, so $1 - x^2 + x^4 \sim x^4$

As $x \to 0$, $\cancel{x}$ " , so $1 - x^2 + x^4 \sim 1$

As $x \to -\infty$ again $1 - x^2 + x^4 \sim x^4$



near 0, $1 - x^2 + x^4 < 1$

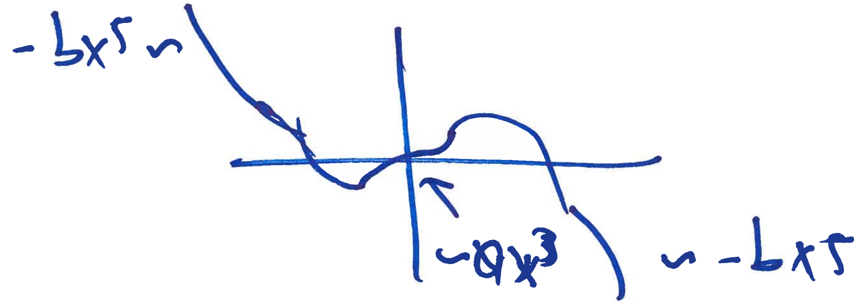
since $x^2 \gg x^4$

(b) $ax^3 - bx^5$ $(a, b > 0)$

As $x \to \infty$, $ax^3 - bx^5 \sim -bx^5$

$x \to 0$, $ax^3 - bx^5 \sim ax^3$

$x \to -\infty$ $ax^3 - bx^5 \sim -bx^5$



Asymptotics of expressions

1. When adding functions, only dominant part(s) matter for overall asymptotics

2. When multiplying, can multiply asymptotics