6. Inverse trig (9/10/2024)

Notes.

(1) WW5 deadline has been extended due to midterm.
(2) Group Project 2 is live.

Goals.

(1) Inverse trig functions
(2) (Differentiation: upcoming small class)

Last Time. Chain rule: $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$

$\rightsquigarrow (f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

Example: $f = \log y$: $(\log g)' = \frac{1}{g} \cdot g' \Rightarrow g' = g \cdot (\log g)'$

Example: Diff along curves / implicit diff: if $y = y(x)$ can diff $F(x,y)$ wrt using chain rule. (even solve for y' if $F(x,y) = 0$)

Inverse functions

$y = f(x)$ answers question "given x what is $f(x)$?"

Also can ask an equation for x, if given y.

Example: domain {students} → $f(\text{student}) = \text{student number}$

domain {valid student #s} → $f^{-1}(\text{student number}) = \text{corresponding student}$

f^{-1} is a function because f is 1:1

ie. different students have different #s

Example: $y = x^2$ on domain $(-\infty, \infty)$

not invertible: equation $4 = x^2$ has two solutions

But $y = x^2$ for $0 \le x < \infty$ ← restricted domain

is invertible

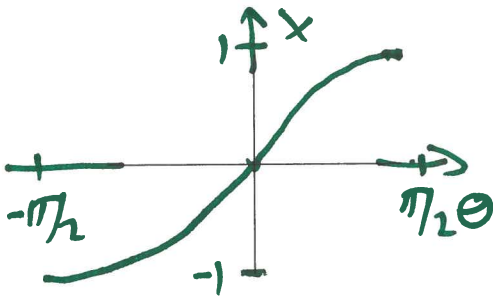
x^2 has range $[0, \infty)$

so $\sqrt{y}$ has domain $[0, \infty)$ ←

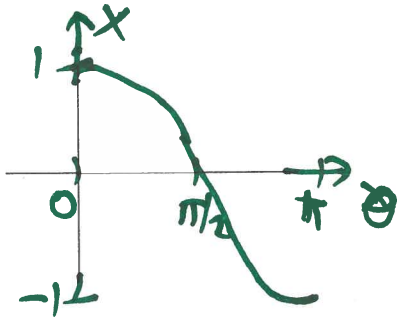
range $[0, \infty)$ ↑

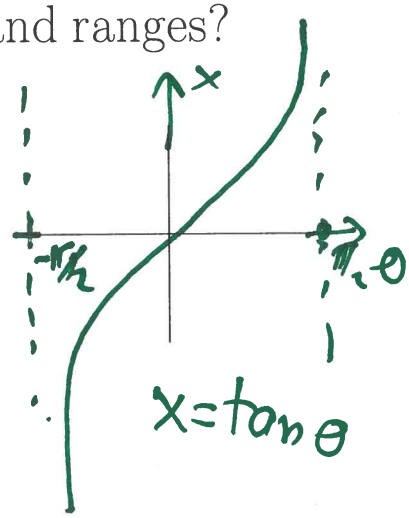
2. Inverse trig functions

(7) Draw on the following axes graphs of $\sin\theta$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\cos\theta$ on $[0, \pi]$ and $\tan\theta$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then of their inverse functions. What are their domains and ranges?

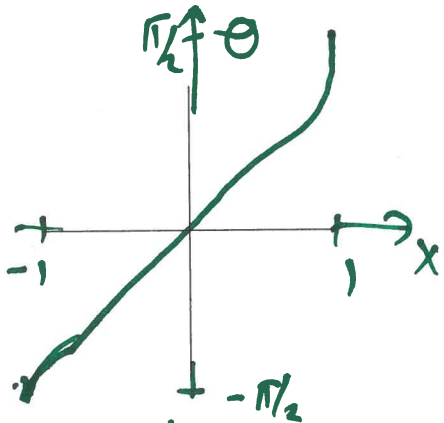


$x = \sin\theta$

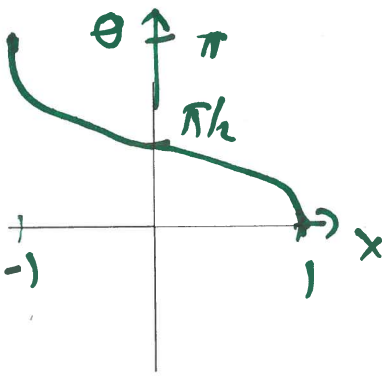




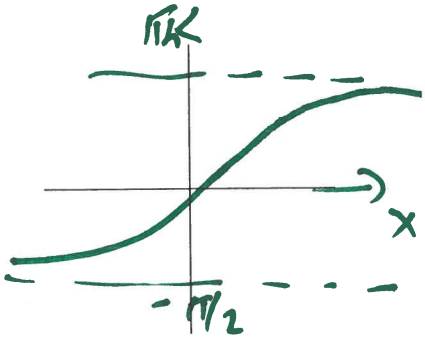
$x = \tan\theta$



$\theta = \arcsin x$



$\theta = \arccos x$



$\theta = \arctan x$

domain $(-\infty, \infty)$

$\lim_{x \to \infty} \arctan x = \pi/2$

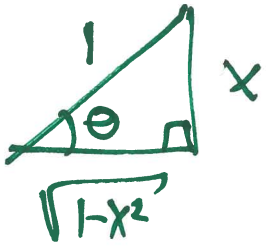
$\lim_{x \to -\infty} \arctan = -\pi$

Idea

say $\theta = \arcsin x$

$\Rightarrow x = \sin\theta$

draw triangle



① set one side to x other side to 1

② compute 3rd side

③ read off other trig fcns

$$\tan(\arcsin x) = \tan\theta = \frac{x}{\sqrt{1-x^2}}$$