Math 100A – WORKSHEET 2
LIMITS. ASYMPTOTES, AND CONTIJUITY

(1) Review of asymptotics: analyze the expression $\frac{e^x + A\sin x}{e^x - x^2}$ as $x \to \infty$, $x \to 0$, $x \to -\infty$.

1. Limits

(2) Either evaluate the limit or explain why it does not exist. Sketching a graph might be helpful.
   (a) $\lim_{x\to 5}\left(x^3 - x\right)$

   (b) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 3 & x = 1 \\ 2 - x^2 & x > 1 \end{cases}$.

   (c) $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} \sqrt{x} & 0 \le x < 1 \\ 1 & x = 1 \\ 4 - x^2 & x > 1 \end{cases}$.

(3) Let $f(x) = \frac{x-3}{x^2+x-12}$.
   (a) (Final 2014) What is $\lim_{x\to 3} f(x)$?

   (b) What about $\lim_{x\to -4} f(x)$?

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(4) Evaluate

(a) $\lim_{x\to\infty} \frac{e^x+A\sin x}{e^x-x^2}$

(b) $\lim_{x\to 0} \frac{e^x+A\sin x}{e^x-x^2}$

(c) $\lim_{x\to-\infty} \frac{e^x+A\sin x}{e^x-x^2}$

(5) Evaluate

(a) $\lim_{x\to 2} \frac{x+1}{4x^2-1}$

(b) (Final, 2014) $\lim_{x\to -3^+} \frac{x+2}{x+3}$.

(c) $\lim_{x\to 1} \frac{e^x(x-1)}{x^2+x-2}$

(d) $\lim_{x\to -2^-} \frac{e^x(x-1)}{x^2+x-2}$

(e) $\lim_{x\to 1} \frac{1}{(x-1)^2}$

(f) $\lim_{x\to 4} \frac{\sin x}{|x-4|}$

(g) $\lim_{x\to \frac{\pi}{2}+} \tan x$, $\lim_{x\to \frac{\pi}{2}-} \tan x$.

2. Asymptotes

(6) For each expression, determine its vertical and horizontal asymptotes.
   (a) $\frac{x^2+1}{x-3}$

   (b) (Final, 2015) $\frac{x+1}{x^2+2x-8}$

(7) (Quiz, 2015) Evaluate $\lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2+x}-2x}$

3. Continuity

(8) Determine where each expression/function is continuous.

(a) $f(x) = \frac{x}{x}$. Can you "fix" the problem at 0?

(b) ("Heaviside step function") $H(x) = \begin{cases} 1 & x > 0 \\ 1/2 & x = 0 \\ 0 & x < 0 \end{cases}$. Can you "fix" the problem at 0?

(c) $g(x) = \sqrt{\log x}$

(9) ("Gluing functions") In each problem find the value of the constant k such that the function is continuous.

(a) $f(x) = \begin{cases} \frac{x^3-2x^2}{x-2} & x \neq 2 \\ k & x = 2 \end{cases}$

(b) $g(x) = \begin{cases} 8 - kx & x < k \\ x^2 & x \geq k \end{cases}$

(c) $h(x) = \begin{cases} Ax^2 + Bx & x \leq k \\ Akx + D & x > k \end{cases}$ (here A, B, D are constants with $B \neq 0$)

(d) $j(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x > 0 \\ k + \cos x & x \leq 0 \end{cases}$