Math 100A – WORKSHEET 3
THE DERIVATIVE

1. Three views of the derivative

(1) Let $f(x) = x^2$, and let $a = 2$. Then $(2, 4)$ is a point on the graph of $y = f(x)$.
   (a) Let (x, x^2) be another point on the graph, close to $(2, 4)$. What is the slope of the line connecting the two? What is the limit of the slopes as $x \to 2$?

   (b) Let h be a small quantity. What is the asymptotic behaviour of $f(2 + h)$ as $h \to 0$? What about $f(2 + h) - f(2)$?

   (c) What is $\lim_{h\to 0} \frac{(2+h)^2-2^2}{h}$?

   (d) What is the equation of the line tangent to the graph of $y = f(x)$ at $(2, 4)$?

(2) An enzymatic reaction occurs at rate $k(T) = T(40 - T) + 10T$ where T is the temperature in degrees celsius. The current temperature of the solution is 20°C. Should we increase or decrease the temperature to increase the reaction rate?

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2. Definition of the derivative

Definition. $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ or $f(a+h) \approx f(a) + f'(a)h$

(3) Use a definition of the derivative to find $f'(a)$ if

(a) $f(x) = x^2$, $a = 3$.

(b) $f(x) = \frac{1}{x}$, any a.

(c) $f(x) = x^3 - 2x$, any a (you may use $(a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$).

(4) Express the limits as derivatives: $\lim_{h\to 0} \frac{\cos(5+h)-\cos 5}{h}$, $\lim_{x\to 0} \frac{\sin x}{x}$

(5) (Final, 2015, variant – gluing derivatives) Is the function

$$f(x) = \begin{cases} x^2 & x \le 0 \\ x^2 \cos \frac{1}{x} & x > 0 \end{cases}$$

differentiable at $x = 0$?

Fact. *The derivative of x^n with respect to x is nx^{n-1}.*

3. The tangent line

Definition. The *line tangent to the graph $y = f(x)$ at $x = a$* is the line $y = f'(a)(x - a) + f(a)$

(6) (Final, 2015) Find the equation of the line tangent to the function $f(x) = \sqrt{x}$ at $(4, 2)$.

(7) (Final 2015) The line $y = 4x + 2$ is tangent at $x = 1$ to which function: $x^3 + 2x^2 + 3x$, $x^2 + 3x + 2$, $2\sqrt{x+3} + 2$, $x^3 + x^2 - x$, $x^3 + x + 2$, none of the above?

(8) Find the lines of slope 3 tangent to the curve $y = x^3 + 4x^2 - 8x + 3$.

(9) The line $y = 5x + B$ is tangent to the curve $y = x^3 + 2x$. What is B?

4. Linear approximation

Definition. $f(a+h) \approx f(a) + f'(a)h$

(10) Estimate

(a) $\sqrt{1.2}$

(b) (Final, 2015) $\sqrt{8}$

(c) (Final, 2016) $(26)^{1/3}$