

Math 100A – WORKSHEET 4
COMPUTING DERIVATIVES

1. REVIEW OF THE DERIVATIVE

(1) Expand $f(x+h)$ to linear order in h for the following functions and read the derivative off:

(a) $f(x) = bx$

(b) $g(x) = ax^2$

(c) $h(x) = ax^2 + bx$.

(d) $i(x) = \frac{1}{b+x}$

(e) $j(x) = 4x^4 + 5x$ (hint: use the known linear approximation to $2x^2$)

2. ARITHMETIC OF DERIVATIVES

Fact. $(af + bg)' = af' + bg'$, $(fg)' = f'g + fg'$, $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
 $\frac{d}{dx}x^n = nx^{n-1}$.

(2) Differentiate

(a) $f(x) = 6x^\pi + 2x^e - x^{7/2}$

(b) (Final, 2016) $g(x) = x^2e^x$ (and then also x^ae^x)

(c) (Final, 2016) $h(x) = \frac{x^2+3}{2x-1}$

(d) $\frac{x^2+A}{\sqrt{x}}$

(3) Let $f(x) = \frac{x}{\sqrt{x+A}}$. Given that $f'(4) = \frac{3}{16}$, give a quadratic equation for A .

(4) Suppose that $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$, $g'(1) = 4$.

(a) What are the linear approximations to f and g at $x = 1$? Use them to find the linear approximation to fg at $x = 1$.

(b) Find $(fg)'(1)$ and $\left(\frac{f}{g}\right)'(1)$.

(5) Evaluate

(a) $(x \cdot x)'$ and $(x') \cdot (x')$. What did we learn?

(b) $\left(\frac{x}{x}\right)'$ and $\frac{(x')}{(x')}$. What did we learn?

- (6) The *Lennart–Jones potential* $V(r) = \epsilon \left(\left(\frac{R}{r} \right)^{12} - 2 \left(\frac{R}{r} \right)^6 \right)$ models the electrostatic potential energy of a diatomic molecule. Here $r > 0$ is the distance between the atoms and $\epsilon, R > 0$ are constants.
- (a) What are the asymptotics of $V(r)$ as $r \rightarrow 0$ and as $r \rightarrow \infty$?

(b) Sketch a plot of $V(r)$.

(c) Find the derivative $\frac{dV}{dr}(r) =$

(d) Where is $V(r)$ increasing? decreasing? Find its minimum location and value.