Math 100A – WORKSHEET 7
APPLICATIONS OF THE CHAIN RULE

1. Related Rates 1: differentiation

(1) A particle is moving along the curve $y^2 = x^3 + 2x$. When it passes the point $\left(1, \sqrt{3}\right)$ we have $\frac{\mathrm{d}y}{\mathrm{d}t} = 1$. Find $\frac{\mathrm{d}x}{\mathrm{d}t}$.

(2) Air is pumped into a spherical balloon at the rate of $13\mathrm{cm}^3/\mathrm{s}$. How fast is the radius of the balloon changing when it is 15cm?

(3) If we place an object at distance p from a thin lens, the *lenscrafter's formula* provides that the distance of the image from the lens, q, is determined by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f},$$

where f is the *focal length* of the lens. Suppose that $f = 10\mathrm{cm}$ and $p = 30\mathrm{cm}$. If we move the object away from the lens at the rate of 4cm/s, how fast is the image moving? In which direction is it moving?

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(4) The *ideal gas law* provides that $PV = NkT$ for mass of N particles of an ideal gas, where P is the pressure, V is the volume, T is the temperature, and k is Boltzmann's constant.

(a) Suppose that we heat a mass of gas in a container of fixed volume. Relate the rate of change of the pressure to the rate of change of the temerature.

(b) Suppose that we compress a piston while holding the temperature constant. Relate the rate of change of the pressure of the gas to the rate of change of its volume. Is the pressure increasing or decreasing?

(5) A closed rectangular box has sides of lengths $4, 5, 6$cm. Suppose that the first and second sides are lengthening by $2\frac{\text{cm}}{\text{sec}}$ while the third side is shortening by $3\frac{\text{cm}}{\text{sec}}$.

(a) How fast is the volume changing?

(b) How fast is the surface area changing?

(c) How fast is the main diagonal changing?

2. Related Rates 2: problem-solving

(6) (Final, 2015, variant) A conical tank of water is 6m tall and has radius 1m at the top.

(a) The drain is clogged, and is filling up with rainwater at the rate of $5\text{m}^3/\text{min}$. How fast is the water rising when its height is 5m?

(b) The drain is unclogged and water begins to drain at the rate of $(5+\frac{\pi}{4})\text{m}^3/\text{min}$ (but rain is still falling). At what height is the water falling at the rate of 1m/min?

(c) Repeat the problem with tank upside-down (vertex on top).

(7) (Final, 2019) A 2m tall woman is running at night, moving away from a 6m-tall lamp-post. Her velocity t seconds after leaving the lamp-post is given (in metres per second) by $v(t) = 4 - \sin(2\pi t)$. How quickly is the length of her shadow changing after 3 seconds?

(8) (Final 2018)

(a) Particle A travels with a constant speed of 2 units per minute on the x-axis starting at the point $(4, 0)$ and moving away from the origin, while particle B travels with a constant speed of 1 unit per minute on the y-axis starting at the point $(0, 8)$ and moving towards the origin. Find the rate of change of the distance between the two particles when the distance between the two particles is exactly 10 units.

(b) Same question, but swap the velocities of the particles (particle A moves along the y axis, particle B moves along the x-axis).