Math 100A – WORKSHEET 8
CURVE SKETCHING

1. Convexity and Concavity

(1) Consider the curve $y = x^3 - x$.
   (a) Find the line tangent to the curve at $x = 1$.

   (b) Near $x = 1$, is the line above or below the curve? Hint: how does the slope of the curve behave to the right and left of the point?

(2) For each curve find its domain; where is it concave up or down? Where are the inflection points.
   (a) $y = x \log x - \frac{1}{2}x^2$.

   (b) $y = \sqrt[3]{x}$.

2. Curve sketching

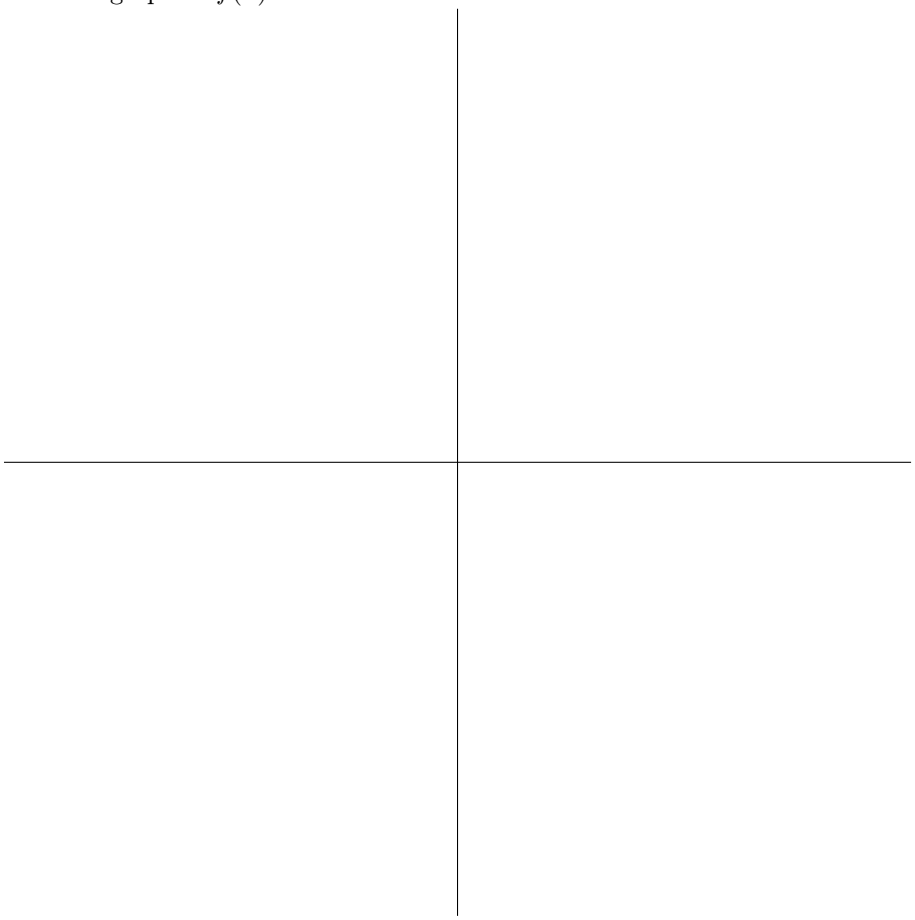
(3) Let $f(x) = \frac{x^2}{x^2+1}$ for which $f'(x) = \frac{2x}{(x^2+1)^2}$ and $f''(x) = \frac{2(1-3x^2)}{(x^2+1)^3}$.
   (a) What are the domain and intercepts of f? What are the asymptotics at $\pm\infty$? Are there any vertical asymptotes? What are the asymptotics there?

   (b) What are the intervals of increase/decrease? The local and global extrema?

Date: 23/10/2024, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

(c) What are the intervals of concavity? Any inflection points?

(d) Sketch a graph of $f(x)$.



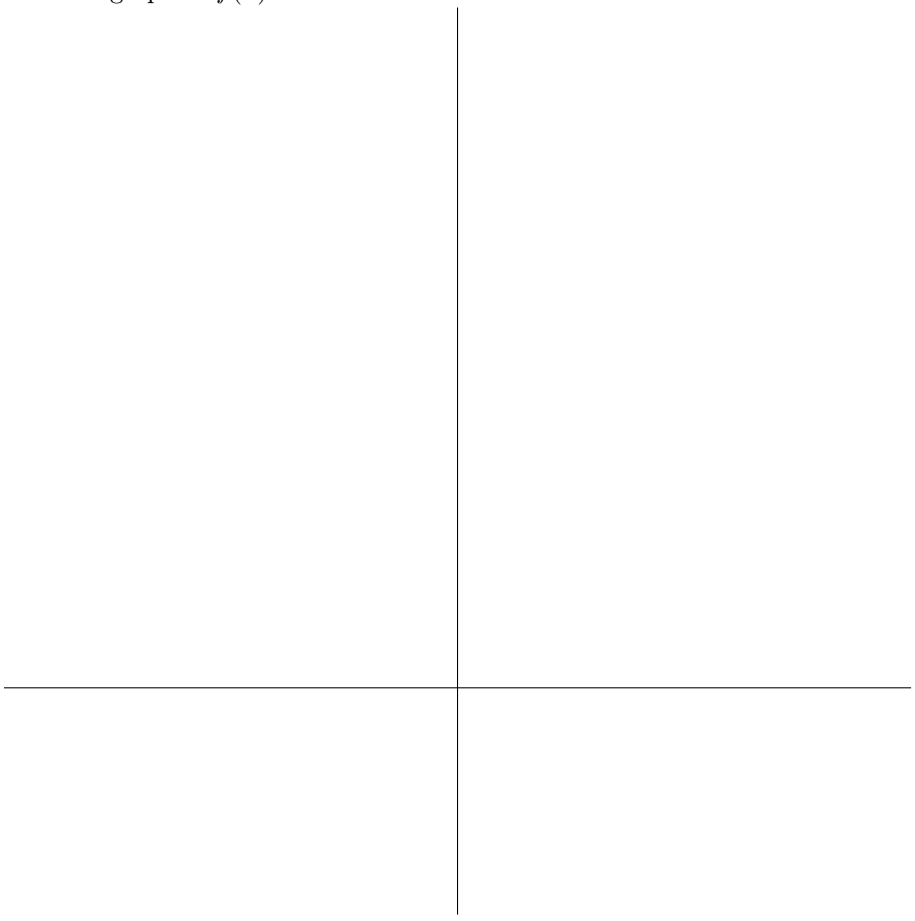
(4) Let $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ for which $f'(x) = -\frac{1}{\sqrt{2\pi\sigma^6}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} (x-\mu)$ and $f''(x) = \frac{1}{\sqrt{2\pi\sigma^6}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left(\frac{(x-\mu)^2}{\sigma^2} - 1 \right)$.

(a) What are the domain and intercepts of f? What are the asymptotics at $\pm\infty$? Are there any vertical asymptotes? What are the asymptotics there?

(b) What are the intervals of increase/decrease? The local and global extrema?

(c) What are the intervals of concavity? Any inflection points?

(d) Sketch a graph of $f(x)$.



(5) (Final, December 2007) Let $f(x) = x\sqrt{3-x}$.

(a) Find its domain, intercepts, and asymptotics at the endpoints.

(b) What are the intervals of increase/decrease? The local and global extrema?

(c) Given $f''(x) = \frac{3x-12}{4}(3-x)^{-3/2}$, what are the intervals of concavity? Any inflection points?

(d) Sketch a graph of $f(x)$.