Math 428, Lecture 1, 7/1/2025

information is on course web page
https://www.math.ubc.ca/~lior

Today: (1) Introduction
(2) Kinematics

(later: (3) Newtonian mechanics
(4) Lagrangian mechanics
(5) Rigid bodies & rotations
(6) Hamiltonian mechanics)

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(1) Kinematics.

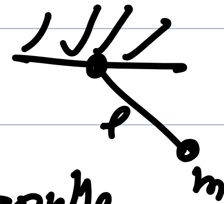
We begin by describing motion
Keyword: configuration space.

Def: A mechanical system consists of several (1) point particles moving in an ambient space (2) subject to interactions (3) and constraints (4)

(2) Almost always, the "ambient space" will be $\mathbb{E}^1$, $\mathbb{E}^2$, $\mathbb{E}^3$, in general $\mathbb{E}^d$,
distinguish between points of affine space $\mathbb{E}^d$
& displacements of the vector space $\mathbb{R}^d$

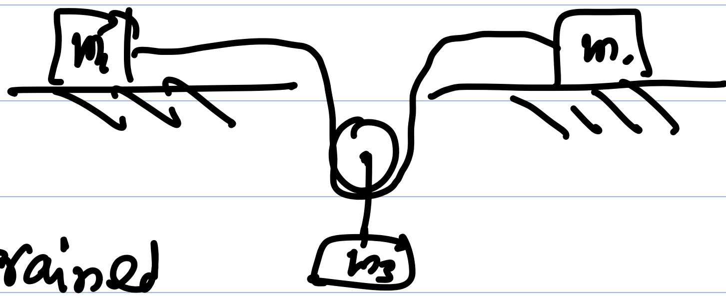
more on affine algebra in PS1

Example: A particle moving on S^2 is moving in $\mathbb{E}^3$, subject to $x^2+y^2+z^2=1$.

Example: 1 particle in $\mathbb{E}^d$, pendulum: 

$\subseteq \mathbb{E}^2$

3d pendulum moving on $S^2 \subset \mathbb{E}^3$, rope-and-pulley,



m_1, m_2 constrained to move $\leftrightarrow$, m_3 constrained to move $\updownarrow$
rope has fixed length.

Remarks: ① From math pov (can) dispense with $\mathbb{E}^d$, just work on config manifold.

② truly ∞-dim systems studied in <u>continuum mechanics</u>

N = # of particles

Def: A <u>configuration</u> of the system is a point $x \in (\mathbb{E}^d)^N$ satisfying the constraints.

<u>configuration space</u> is the set X of configurations.

(*) constraints: include holonomic constraints

locally of the form $F(x)=0$, $F:\mathbb{R}^{dN}\to\mathbb{R}^m$

(e.g. motion on sphere)

but also boundaries (e.g. particle in a box)

constraints on motion

Pre-requisites: (1) Newtonian mechanics
(say UBC PHYS 216)

(2) Linear algebra (say UBC 223
221)

(3) Real analysis (say UBC 320)

Q: Semiclassical limits

$a \in C^\infty(\text{phase space}) \rightsquigarrow Op_\hbar(a)$ - quantum observable

$\hat{H} = Op_\hbar(H)$, $g_t : X \to X$ Hamiltonian flow.

$U(t) = \exp\left(\frac{it\hat{H}}{\hbar}\right)$ quantum propagator.

Thm: (Egorov) $\hat{a}(t) = U(-t)\, Op_\hbar(a)\, U(t)$

$$= Op_\hbar(a \circ g_t) + O(\hbar)$$