2011 Calculus Challenge Examination — Comments

[10] **1.** The differentiation questions were done well. The antiderivative questions were not. Most students tried some polynomial power rule, or tried to do both using logarithms, or tried to do both using arctan.

[9] **2.** (a) Well done. About half the students used the factor-and-cancel approach suggested in the online solutions; the others used L'Hopital's rule.

(b) Well done. There was a reasonably even split between students multiplying by the conjugate, etc., and strudents using l'Hopital's rule.

(c) Very poorly done. Few students realized that the limit would depend on a. Most students treated the case $a > 1$ implicitly, i.e., they claimed that $\lim a^x = +\infty$ and concluded that the limit was equal to 1. Most of the students that realized that the limit depended on a only treated the cases $0 < a < 1$ and $a > 1$. They missed the case $a = 1$, which they mistakenly lumped together with one of the other two.

[10] **3.** Well done. Most students got part (a) perfectly. A large portion of the students did not include a sketch for part (b). Most errors in this question were computational.

[5] **4.** Generally well done. Almost everybody got the derivative correct. For the tangent line approximation, i.e., $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$, a common mistake was to select x_0 from the table instead of consistently using $x_0 = 0$.

[5] **5.** Mixed results. Those who recognized the differential equation and could recall its solution had no problem. A few students even managed to derive the solution on their own. Roughly half of the students fit into these two categories. The other half didn't have a place to begin and the rest of their solution suffered.

[8] **6.** (a) Generally well done. No problem for the first derivative. Some students concluded a bit fast about the concavity of y, i.e., they did not simplify y'' so that the sign of y'' was not obvious. Sketch was generally VERY rough.

(b) Most students who got started used the approach in the online solutions and got the right answer. Unfortunately, only a minority made a good start.

[8] **7.** Parts (a) and (b) were handled with little difficulty by most students. Part (c) was hit or miss. Again, if a student knew about L'Hopital, the question went rather smoothly. Otherwise, the solution went all over the place. Computing the derivatives themselves posed very few problems.

[6] **8.** (a) Generally well done, although some complicated examples were given, e.g., $y = \cos(x)\sin(x)$ whereas x would suffice.

(b) Mixed results. The identity $(y^2)' = 2yy'$ was obtained by most of the students. Dividing by y' led them to the equation $y' = 2y$, but most of them forgot about the possibility that $y' = 0$. They finally ended up with having only the exponential for the solution.

[6] **9.** Well done, mostly. Problems with this question (amongst those who completed the question—some writers stopped at the midway point for no discernible reason) had to do with solving for constants and factoring to find the time T. Most likely silly computational mistakes caused by stress. Most knew how to solve the problem, even if they made a mistake along the way.

[8] **10.** Mixed results. Eveybody that got the question correct did the calculation for the three cases and concluded afterwards that the speed was equal for these three cases. Nobody mentioned up front that the speed would always be the same. A lot of students obtained an identity using similar triangles, but stopped there or got confused by what was to be calculated. Some students started heavy trigonometric calculations that must have cost them a lot of time and ended up nowhere. Finally, the sketch of the situation—so important in problems like this—was generally poor.

[9] **11.** The most common answers were $\sqrt{2}$, $\sqrt{2/3}$, and sometimes 1. Of these, $\sqrt{2/3}$ is the closest to the correct answer of $\sqrt{2/3}\,L$, but does not take into account that $L = 3$. As usual, the biggest difficulty was in the translation from a bunch of words into the correct function to be maximized.

[9] **12.** Generally well done. Mostly, the errors came from mistakes in the calculation of the second derivative. Some students also concluded a bit fast about the sign of y, i.e., they did not simplify y'' to a point where the sign of y'' was evident. Surprisingly, not everybody used the axes provided in the exam. Generally, the sketch of the function was acceptable, but more information could have been provided, i.e., asymptote, location of the inflection points, etc.

[7] **13.** Well done. The most common error was splitting the area into three parts instead of two, using the point where one of the functions crossed the x-axis to trigger an (unneccessary, incorrect) additional change in the order of the integrands.