UBC Mathematics 402(201)—Assignment 1
Due by PDF upload to Canvas no later than 23:59, Friday 16 January 2026

Please watch Canvas or listen in class for details on formatting and submission standards.

1. For each Lagrangian below, write the Euler-Lagrange equation (DEL) and find all C^2 solutions. (The parameters ω and k are constant.)

(a) $L(t, x, v) = v^2 - \omega^2 x^2$, $\omega > 0$,

(b) $L(t, x, v) = v^2 + k^2 x^2$, $k > 0$,

(c) $L(t, x, v) = v^2 + x^2 - 4(\sin t)x$,

(d) $L(t, x, v) = v^2 - 6t^2 x$,

(e) $L(t, x, v) = (v - x)^2 + 2e^{kt}x$, $k^2 \neq 1$.

2. Consider this instance of the Basic Problem:

$$\begin{aligned} &\text{minimize} && \Lambda[x] = \int_0^1 \left(\dot{x}(t)^2 + x(t)^2 + 4e^t x(t)\right) dt \\ &\text{over all} && x \in C^2[0,1] \\ &\text{subject to} && x(0) = 0,\ x(1) = e^{-1}. \end{aligned}$$

Identify the unique global minimizer and find its Λ-value. Provide careful justification for the property of global minimality.

3. Consider the following variational problem, for which the arc $\widehat{x}(t) = t$ is admissible:

$$\min \left\{ \Lambda[x] \stackrel{\text{def}}{=} \int_1^3 t(\dot{x}^2(t) - x^2(t))\, dt \ :\ x(1) = 1,\ x(3) = 3 \right\}. \qquad (P)$$

Use whatever software you choose (including "none"), to help complete the activities below.

(a) One admissible variation is $h_1(t) = (t-1)(t-3)$. Find the [quadratic] function

$$q(\lambda) = \Lambda[\widehat{x} + \lambda h_1]$$

and sketch its graph. Then find the λ-value that minimizes q and the corresponding arc $x = \widehat{x} + \lambda h_1$.

(b) Imagine using a variation built from two ingredients, each with its own scale factor. To be specific, keep h_1 from part (a), invent $h_2(t) = (t-1)(t-2)(t-3)$, and consider the 2-parameter family of admissible arcs

$$x(t; \lambda_1, \lambda_2) = \widehat{x}(t) + \lambda_1 h_1(t) + \lambda_2 h_2(t), \qquad 1 \leq t \leq 3.$$

Let

$$f(\lambda_1, \lambda_2) = \Lambda[x(\cdot; \lambda_1, \lambda_2)].$$

Write this [quadratic] function explicitly, and sketch its graph. Then find the point (λ_1, λ_2) that minimizes f and the corresponding arc x.

(c) On the same set of axes, sketch the reference arc and the improvements found in parts (a) and (b). Calculate and compare the Λ-values for these three arcs.

Please note: If you opt for software assistance, please ...

- Report all inexact (computed) values with five significant figures,
- Include enough computer output to enable someone of modest skills to reproduce your work,
- Organize your submission so the answers for items (a)–(c) above are easy to find.