Multivariable Calculus - Math 253, Section 102

Fall 2006

Section 15.5

4.

$$\frac{dz}{dt} = \left[\frac{x}{x+2y} + \ln(x+2y)\right]\cos t - \frac{2x}{x+2y}\sin t$$

6.

$$\frac{dw}{dt} = e^t\left[(y+(x+z^2)(\cos t + \sin t) + 2yz(\cos t - \sin t)\right].$$

8.

$$\frac{\partial z}{\partial s} = \frac{1}{y}e^t - \frac{x}{y^2}e^{-t}$$
$$\frac{\partial z}{\partial t} = \frac{s}{y}e^t + \frac{xs}{y^2}e^{-t}.$$

14. $W_s(1,0) = 52$

22.

$$\frac{\partial u}{\partial x} = \frac{r\cos t + s}{\sqrt{r^2+s^2}},$$
$$\frac{\partial u}{\partial y} = \frac{r + s\sin t}{\sqrt{r^2+s^2}},$$
$$\frac{\partial u}{\partial t} = \frac{-rx\sin t + sy\cos t}{\sqrt{r^2+s^2}}.$$

24.

$$\frac{\partial M}{\partial u} = 16, \quad \frac{\partial M}{\partial v} = 36.$$

36. (b) Production will decrease at the rate of 1.1 units/year.

38. $8160\ \pi$ in^3/s.

40. $dI/dt = -0.000031$ A/s.

42. -118 km/h.

50.

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\sin\theta,$$

$$\frac{\partial z}{\partial \theta} = -\frac{\partial z}{\partial x} r\sin\theta + \frac{\partial z}{\partial y} r\cos\theta,$$

$$\frac{\partial^2 z}{\partial r \partial \theta} = \cos\theta\frac{\partial z}{\partial y} - \sin\theta\frac{\partial z}{\partial x} + r\cos\theta\sin\theta\left(\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2}\right) + r(\cos^2\theta - \sin^2\theta)\frac{\partial^2 z}{\partial y \partial x}.$$

Section 15.6

8. $12/5$.

14. $-\frac{3\sqrt{3}+2}{2\sqrt{13}}$.

16. $-\frac{9}{2\sqrt{14}}$.

22. Direction of maximum rate of change $= \langle 1, 1 \rangle$. The maximum rate $= \sqrt{2}$.

24. Maximum rate of change is $\sqrt{29}$ in the direction $(2, 3, 4)$.

28. ≈ 5.64.

30. 3.92.

32.

(a) $-\frac{5200\sqrt{6}}{3e^{43}}$ °C/m.
(b) $\langle -2, 3, 18 \rangle$.
(c) $400e^{-43}\sqrt{337}$ °C/m.

34.

(a) ascend at the rate of 3.2 vertical meters per horizontal meter.
(b) descend at the rate of approximately 1.56 vertical meters per horizontal meter.
(c) $\langle -1, -3.2 \rangle$ is the direction of largest slope. Angle above the horizontal is $\approx 73.4°$.

40.

(a) $-x + 2y = 3$,

(b)

$$\frac{x+1}{-1} = \frac{y-1}{2}, \quad z = 0.$$

48. $x + 2y = 1$