Multivariable Calculus - Math 253, Section 102

Fall 2006

Section 15.7

8. The only critical point is $(0, 2)$. The value $f(0, 2) = e^4$ is a local maximum.

12. The critical points are $(0, 0), (1, 0), (0, 1)$ and $(\frac{1}{3}, \frac{1}{3})$. Among these $(0, 0), (1, 0), (0, 1)$ are saddle points, and $f(\frac{1}{3}, \frac{1}{3}) = \frac{1}{27}$ is a local maximum.

28. The absolute maximum of f on D is $f(1, 0) = f(3, 2) = 2$ and the absolute minimum is $f(1, 4) = f(5, 0) = -2$.

30. The absolute maximum is $f(2, 3) = 13$ and the absolute minimum is attained at both $(0, 0)$ and $(4, 0)$, where $f(0, 0) = f(4, 0) = 0$.

32. The absolute maximum of f on D is $f(1, \sqrt{2}) = 2$, and the absolute minimum is 0 which occurs at all points along the line segments $L_1 = \{(x, 0) : 0 \leq x \leq \sqrt{3}\}$, and $L_2 = \{(0, y) : 0 \leq y \leq \sqrt{3}\}$.

38. The point on the plane closest to $(1, 2, 3)$ is $(\frac{5}{3}, \frac{4}{3}, \frac{11}{3})$.

42. The maximum occurs when $x = 100a/(a+b+c)$ and $y = 100b/(a + b + c)$.

48. The dimensions of the aquarium that minimize the cost are $x = y = (\frac{2}{5}V)^{1/3}$ units, $z = V^{\frac{1}{3}}(\frac{5}{2})^{2/3}$.

Section 15.8

4. The constrained maximum of f is $f(2,3) = 26$ and the constrained minimum is $f(-2,-3) = -26$.

8. The constrained maximum of f is $f(2,0,-1) = 20$ and the constrained minimum is $f(-2,0,1) = -20$.

18. The constrained maximum of f is $f(-2, \pm 2\sqrt{3}) = 47$ and the constrained minimum is $f(1,0) = -7$.

38. The minimum and maximum of f are respectively

$$f\left(\frac{1}{3}(50 - 10\sqrt{3}, \frac{1}{3}(50 + 5\sqrt{10}), \frac{1}{3}(50 + 5\sqrt{10}))\right) = \frac{1}{27}(87,500 - 2500\sqrt{10}),$$

$$f\left(\frac{1}{3}(50 + 10\sqrt{3}, \frac{1}{3}(50 - 5\sqrt{10}), \frac{1}{3}(50 - 5\sqrt{10}))\right) = \frac{1}{27}(87,500 + 2500\sqrt{10}).$$