Math 253, Section 102, Fall 2006
Practice Final

1. Determine whether the two lines L_1 and L_2 described below intersect. If yes, find the point of intersection. If not, say whether they are parallel or skew, and find the shortest distance between them. The line L_1 is described by the equations

$$x - 1 = 2y + 2, \quad z = 4,$$

and the line L_2 passes through the points $P(2, 1, -3)$ and $Q(0, 8, 4)$.

2. Find and sketch the largest possible domain of the function

$$f(x, y, z) = \arcsin(3 - x^2 - y^2 - z^2).$$

3. Find the equation of the tangent plane to the surface

$$yz = \ln(x + z)$$

at the point $(1, 0, 0)$.

4. The plane $4x + 9y + z = 0$ intersects the elliptic paraboloid $z = 2x^2 + 3y^2$ in an ellipse. Find the highest and lowest points on this ellipse.

5. Find the area that is cut from the surface $z = x^2 - y^2$ by the cylinder $x^2 + y^2 = 4$.

6. You are standing at the point where $x = y = 100$ feet on the side of a mountain whose height (in feet) above the sea level is given by

$$z = f(x, y) = \frac{1}{1000}(3x^2 - 5xy + y^2),$$

with the x-axis pointing east and the y-axis pointing north.

   (a) If you head northeast, will you be ascending or descending? How fast?

   (b) In which direction should you head in order to descend the fastest?

   (c) Suppose that you decide to move in a direction that makes an angle of $45°$ with $(1, -3)$. How fast will you be ascending or descending then?

(d) In which direction should you be moving in order to remain at the same altitude?

7. Compute the value of the triple integral
$$\iiint_E z\,dV,$$
where E is the region between the surfaces $z = y^2$ and $z = 8 - y^2$ for $-1 \leq x \leq 1$.

8. According to van der Waal's equation, 1 mol of gas satisfies the equation
$$\left(p + \frac{a}{V^2}\right)(V - b) = cT$$
where p, V and T denote pressure (in atm), volume (in cm^3) and temperature (in kelvins) respectively, and a, b, c are constants. Suppose there exists a gas of volume 2 cm^3, pressure 1 atm and at temperature 5K for which $a = 16$, $b = c = 1$. Use differentials to approximate the change in its volume if p is increased 2 atm and T is increased to 8K.

9. Evaluate
$$\iiint_E xyz\,dV$$
where E lies between the spheres $\rho = 2$ and $\rho = 4$ and above the cone $\phi = \pi/3$. Here ρ and ϕ have the same interpretation as in spherical coordinates.

10. Identify all the local maximum, minimum and saddle points of the function
$$f(x, y) = (x^2 + y)e^{\frac{y}{2}}.$$