# Math 253, Section 102, Fall 2006
# Midterm, October 25

## Name: SID:

### Instructions

- The total time is 50 minutes.
- The total score is 50 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- Calculators and cheat sheets are not allowed.

| **Problem** | **Points** | **Score** |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 10 | |
| 4 | 10 | |
| **TOTAL** | 50 | |

1. Give brief answers to each of the following questions. Please show all your work leading up to the answer.

   $(2 + 4 + 5 + 4 = 15 \text{ points})$

   (a) Find the angle between the vectors $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

   (b) Do the following lines intersect? If yes, find the point of intersection. If not, explain why not.

$$\begin{cases} x &= 1 + t \\ y &= 1 - t \\ z &= 2t \end{cases} \quad \text{and} \quad \begin{cases} x &= 2 - s \\ y &= s \\ z &= 2. \end{cases}$$

(c) You are given the equation $r^2 = r$ in cylindrical coordinates. Describe and sketch the region it represents in 3-space.

(d) You measure the length, width and height of a box to be 10cm, 5cm and 3cm respectively. The scale you used was off by a cm in each measurement. How much error did you make in computing the volume of the box?

2. All the questions in this item are about the following surface :

$$x^2 + y^2 + 2z^2 = 1.$$

(2 + 8 + 5 = 15 points)

(a) Identify and sketch the surface.

(b) At which point(s) of the surface does the normal vector point along the direction of the line

$$\frac{x-1}{\sqrt{2}} = \frac{z}{2}, \quad y = -3?$$

(c) Find the equation of the tangent plane to the surface at the point where $x = \frac{3}{5}$, $y = \frac{4}{5}$.

(d) (5 extra credit points) An ant lies on the surface at the point $x = y = z = \frac{1}{2}$. In which direction should it go in order to climb the steepest slope?

3. For each question in this item, find the limit, or show that the limit does not exist.

(5 + 5 = 10 points)

(a) $$\lim_{(x,y)\to(0,0)} \frac{\tan^2(\sqrt{x^2+y^2})}{x^2+y^2}.$$

(b) $$\lim_{(x,y)\to(0,0)} \frac{x^4y^3}{x^8+y^6}.$$

4. You are given a function $z = f(x, y)$, with

$$f_y(1, 0) = 0, \quad f_{xy}(1, 0) = 1.$$

Introduce two new variables $(r, s)$ that depend on $(x, y)$ as follows

$$x = r^2 + s^2, \quad y = 2rs.$$

Find

$$\frac{\partial z}{\partial r} \text{ and } \frac{\partial^2 z}{\partial s \partial r} \text{ when } r = 0, s = 1.$$

(3 + 7 = 10 points)