Math 253, Section 102, Fall 2006
Practice Midterm

Name: SID:

Instructions

- The total time is 50 minutes.
- The total score is 100 points.
- Use the reverse side of each page if you need extra space.
- Show all your work. A correct answer without intermediate steps will receive no credit.
- Calculators and cheat sheets are not allowed.

Problem	Points	Score
1	15	
2	15	
3	10	
4	10	
5	10	
6	20	
7	20	
TOTAL	25	

1. Prove that the lines

$$x-1=\frac{1}{2}(y+1)=z-2 \quad \text{ and } x-2=\frac{1}{3}(y-2)=\frac{1}{2}(z-4)$$

intersect. Find an equation of the only plane that contains them both.

(7 + 8 = 15 points)

2. For each of the following, either compute the limit or show that the limit does not exist.

(7 + 8 = 15 points)

(a)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + y^4}{(x^2 + y^2)^{\frac{3}{2}}},$$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{2x^3y^2}{x^6 + y^4}.$$

3. There is only one point at which the plane tangent to the surface

$$z = x^2 + 2xy + 2y^2 - 6x + 8y$$

is horizontal. Find it.

(10 points)

4. Identify the surface $x = \sin y$ in (x, y, z)-space, and sketch its graph.

(3 + 7 = 10 points)

5. You buy a giftbox whose dimensions are 10 cm by 15 cm by 20 cm, but there may be a possible error of 0.1 cm in each. You want to buy just enough gift-wrapping paper to fully cover your box. What is the maximum error you should allow for while purchasing the paper?

(10 points)

6. The sun is melting a rectangular block of ice. When the block's height is 1 m and the edge of its square base is 2 m, its height is decreasing at 20 cm/hr and its base edge is decreasing at 30 cm/hr. How fast is the volume of the ice block shrinking at that instant?

(20 points)

7. Suppose that $w = f(x, y)$, $x = r\cos\theta$ and $y = r\sin\theta$. Show that

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2.$$

(20 points)