Math 263 Assignment 5

Due October 10

Problems from the text (do NOT turn in these problems):
(16.2) 3-22, 25-30; (16.3) 1-28, 37-48.

Problems to turn in:

1. The temperature at all points in the disc $x^2 + y^2 \leq 1$ is $T(x, y) = (x + y)e^{-x^2-y^2}$. Find the maximum and minimum temperatures on the disc.

2. Find the high and low points of the surface $z = \sqrt{x^2 + y^2}$ with (x, y) varying over the square $|x| \leq 1, |y| \leq 1$. Discuss the values of z_x, z_y there. Do not evaluate any derivatives in answering this question.

3. Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x + y - z$ on the sphere $x^2 + y^2 + z^2 = 1$.

4. Find a, b and c so that the volume $4\pi abc/3$ of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ passing through the point $(1, 2, 1)$ is as small as possible.

5. Find the triangle of largest area that can be inscribed in the circle $x^2 + y^2 = 1$.

6. For each of the following, evaluate the given double integral **without** using iteration. Instead, interpret the integral as an area or some other physical quantity.
   (a) $\iint_R dx\,dy$ where R is the rectangle $-1 \leq x \leq 3, -4 \leq y \leq 1$.
   (b) $\iint_D (x + 3)dx\,dy$, where D is the half disc $0 \leq y \leq \sqrt{4 - x^2}$.
   (c) $\iint_R (x + y)dx\,dy$ where R is the rectangle $0 \leq x \leq a, 0 \leq y \leq b$.
   (d) $\iint_R \sqrt{a^2 - x^2 - y^2}\,dx\,dy$ where R is the region $x^2 + y^2 \leq a^2$.
   (e) $\iint_R \sqrt{b^2 - y^2}\,dx\,dy$ where R is the rectangle $0 \leq x \leq a, 0 \leq y \leq b$.

7. For each iterated integral, sketch the domain of integration and evaluate:
   (a)
   $$I = \int_0^1 \int_y^1 e^{-x^2}\,dxdy$$
   (b)
   $$I = \int_0^1 \int_x^1 \frac{y^p}{x^2 + y^2}\,dydx \ (p > 0)$$
   (c)
   $$I = \int_0^{\pi/2} \int_y^{\pi/2} \frac{\sin x}{x}\,dxdy$$