Math 263 Assignment 3

Due September 26

Problems from the text (do NOT turn in these problems):
(16.7) 9-12, 17-28. (16.8) 7-14, 21-30, (17.1) 5, 7, 8, 15-18, 31, (17.2) 4-6, 17-22.

Problems to turn in:

1. In each case sketch the region and then compute the volume of the solid region.

   (a) The "ice-cream cone" region which is bounded above by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and below by the cone $z = \sqrt{x^2 + y^2}$.

   (b) The region bounded by $z = x^2 + 3y^2$ and $z = 4 - y^2$.

   (c) A sphere with a cylindrical hole bored through its centre. Specifically, the region inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 4$.

2. Switch these integrals to spherical coordinates and compute:

$$I_1 = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2 + y^2 + z^2}\, dzdydx$$

$$I_2 = \int_{0}^{3} \int_{0}^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2)\, dzdxdy$$

3. Calculate the moment of inertia of a circular pipe of outer radius a, inner radius b, length L and uniform density R, rotating about its centre axis. From your answer, let $b \to 0$ and derive the formula for a solid cylinder too.

4. Find the gradient vector field of $f(x, y) = \sqrt{x^2 + y^2}$ and $g(x, y) = x^2 - y$. In each case, plot the gradient vector field and the contour plot of the function, on the same diagram.

5. Compute $\int_C f(x, y, z)ds$ for the following curves and functions.

   (a) $C_1 : \mathbf{r}(t) = \langle 30\cos^3 t, 30\sin^3 t\rangle$ for $0 \leq t \leq \pi/2$ and $f(x, y) = 1 + y/3$.

   (b) $C_2 : \mathbf{r}(t) = \langle t^2/2, t^3/3\rangle$ for $0 \leq t \leq 1$ and $f(x, y) = x^2 + y^2$.

   (c) $C_3 : \mathbf{r}(t) = \langle 1, 2, t^2\rangle$ for $0 \leq t \leq 1$ and $f(x, y, z) = e^{\sqrt{z}}$.

6. Determine whether or not the following vector fields are conservative. In the cases where $\mathbf{F}$ is conservative, find a function φ such that $\mathbf{F}(x, y, z) = \nabla\varphi(x, y, z)$.

   (a) $\mathbf{F} = (2xy + z^2)\mathbf{i} + (x^2 + 2yz)\mathbf{j} + (y^2 + 2xz)\mathbf{k}$.

   (b) $\mathbf{F} = (\ln(xy))\mathbf{i} + (\frac{x}{y})\mathbf{j} + (y)\mathbf{k}$.

   c) $\mathbf{F} = (e^x \cos y)\mathbf{i} + (-e^x \sin y)\mathbf{j} + (2z)\mathbf{k}$.