

Mathe 263 Practice Problem Set 1

1. Find parametric equations for the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $(-1, 1, 2)$.

(Answer: $x = -1 + 5t, y = 1 + 8t, z = 2 + 6t$)

2. Show that the sum of the x , y and z -intercepts of any tangent plane of the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}$ is a constant.
3. The radius of a right circular cone is increasing at a rate 1.8 in/s, while its height is decreasing at a rate of 2.5 in/s. At what rate is the volume of the cone changing when the radius is 120 inches and the height is 140 inches?

(Answer: 8160π in³/s)

4. Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

(Answer: $\theta = \frac{\pi}{2}, 2\pi - \cos^{-1}(-\frac{8}{17})$)

5. Near a buoy, the depth of a lake at the point with coordinates (x, y) is $z = 200 + 0.02x^2 - 0.001y^3$ where x, y, z are in meters. A fisherman starts at $(80, 60)$ and moves towards the buoy which is located $(0, 0)$. Is the boat getting deeper or shallower when he departs?

(Answer: depth is increasing in the direction toward the buoy)